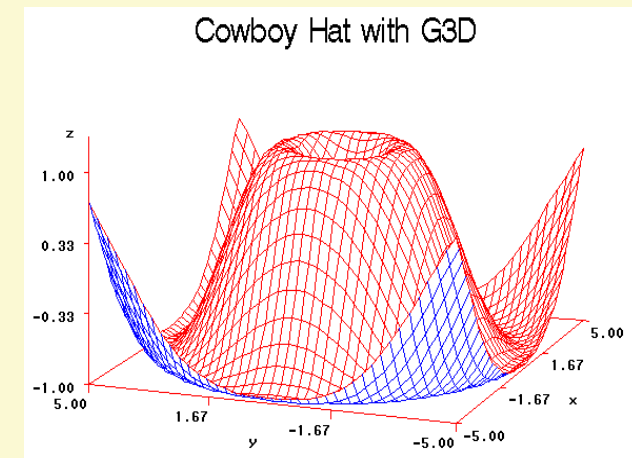


L4: Multiple Linear Regression

Presented by

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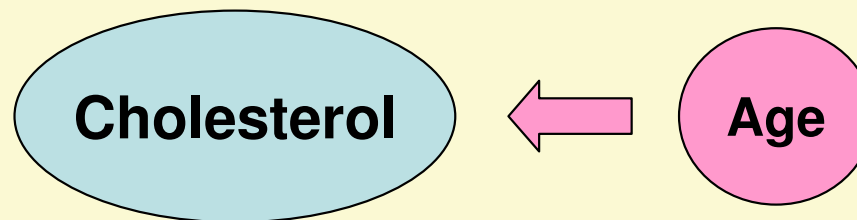
Basic Theory of Multiple Linear Regression

Steps in Handling Multiple Linear Regression Analysis

Data Presentation and Interpretation

Simple Linear Regression

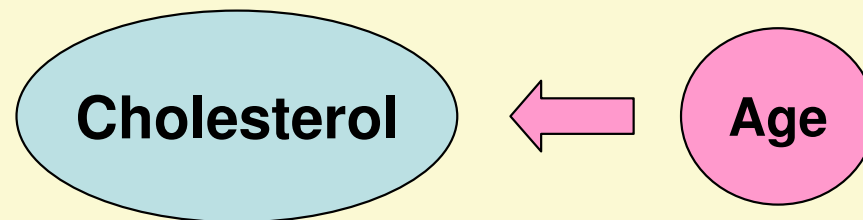
- To determine the relationship between age and blood cholesterol level



- ▶ Here, we may use either 'correlation analysis' or 'regression analysis', as both cholesterol and age are numerical variables.
- ▶ *Correlation* can give the strength of relationship, but *regression* can describe the relationship in more detail.
- ▶ In above example, if we decide to do regression, cholesterol will be our outcome (dependent) variable, because age may determine cholesterol but cholesterol cannot determine age.

Simple Linear Regression

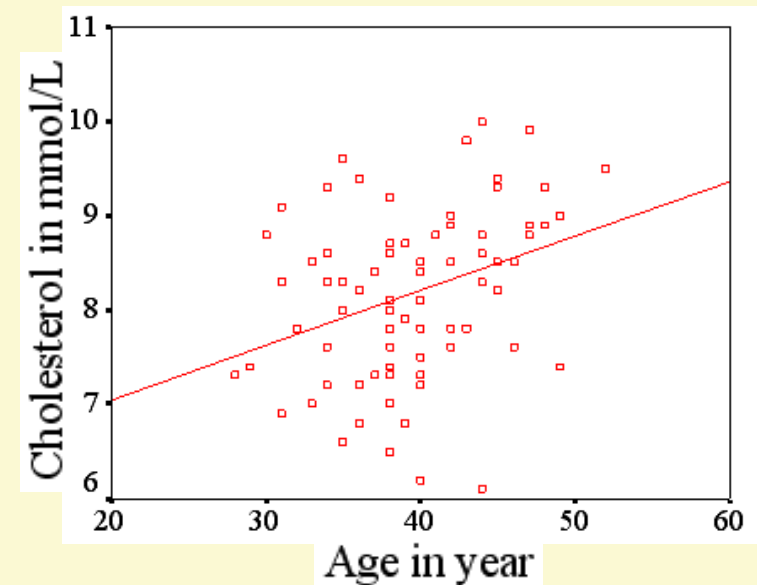
- To determine the relationship between age and blood cholesterol level



Simple Linear Regression

The image shows the Minitab software interface for creating a simple linear regression plot. The 'Graphs' menu is open, and the 'Scatterplot' dialog box is shown. The 'Simple Scatterplot' sub-dialog is also visible, with variables 'chol' and 'age' assigned to the Y and X axes respectively. Numbered callouts 1 through 4 indicate the steps:

1. Selecting 'Scatter...' from the Graphs menu.
2. Clicking the 'Define' button in the Scatterplot dialog.
3. Selecting 'chol' for the Y Axis in the Simple Scatterplot dialog.
4. Selecting 'age' for the X Axis in the Simple Scatterplot dialog.



Simple Linear Regression

Data Editor

Analyze > Regression > Linear...

Linear Regression

Dependent: chol

Independent(s): age

Method: Enter

Linear Regression: Statistics

Regression Coefficients

- Estimates
- Confidence intervals
- Variance matrix

$$Y = a + bX$$

$$\text{Chol} = 5.9 + (0.058 * \text{age})$$

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	P value	95% Confidence Interval for B	
		B	Std. Error	Beta		Sig.	Lower Bound	Upper Bound
1	(Constant)	5.895	.735		8.026	.000	4.434	7.357
	AGE age in year	5.776E-02	.018	.331	3.134	.002	.021	.094

a. Dependent Variable: CHOL cholesterol in mmol/L

Slope (b) = 0.058 (95% CI: 0.021, 0.094)

Cholesterol in mmol/L

Slope (b) = 0.058~ 0.06
mmol/l

1 year older

20

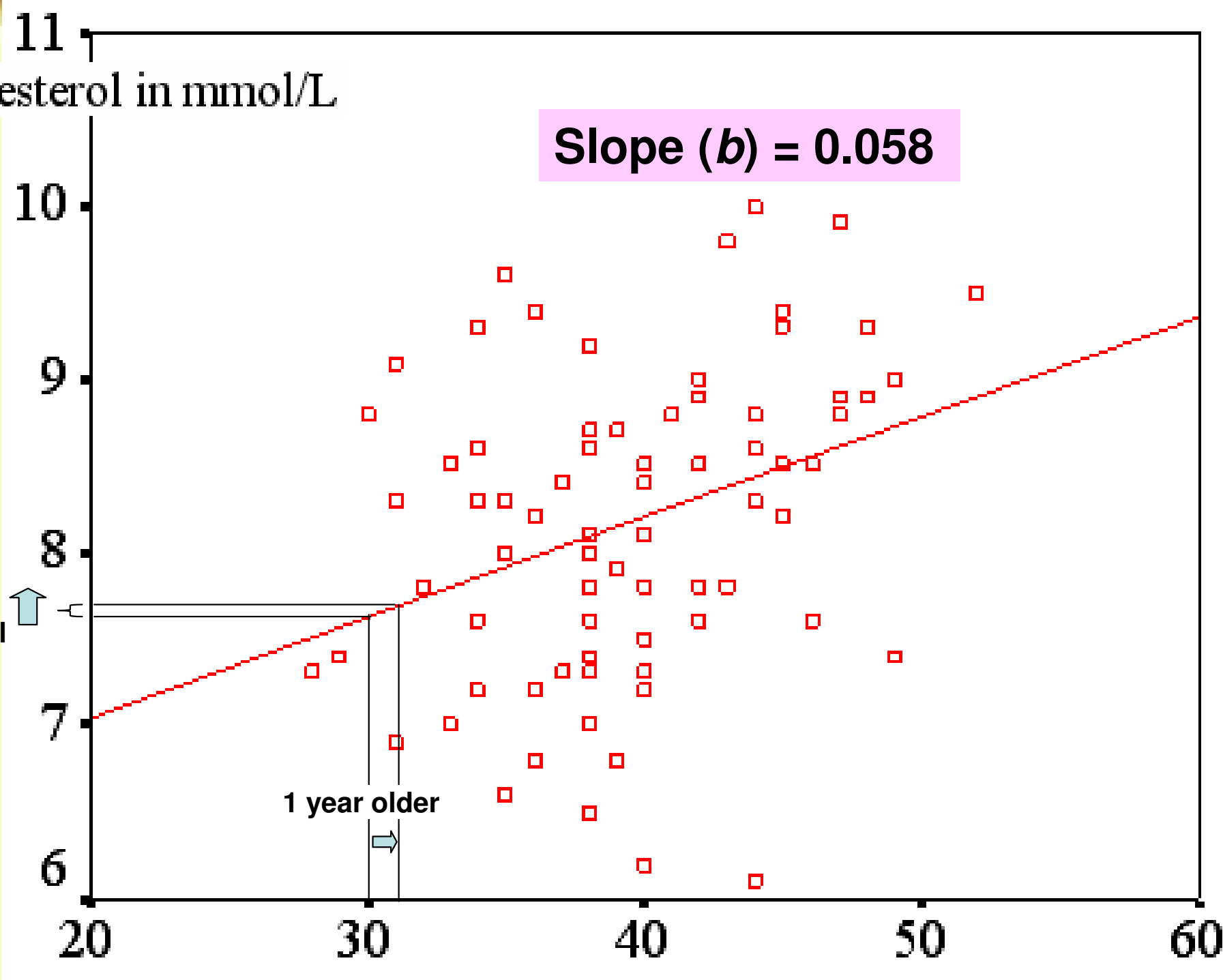
30

40

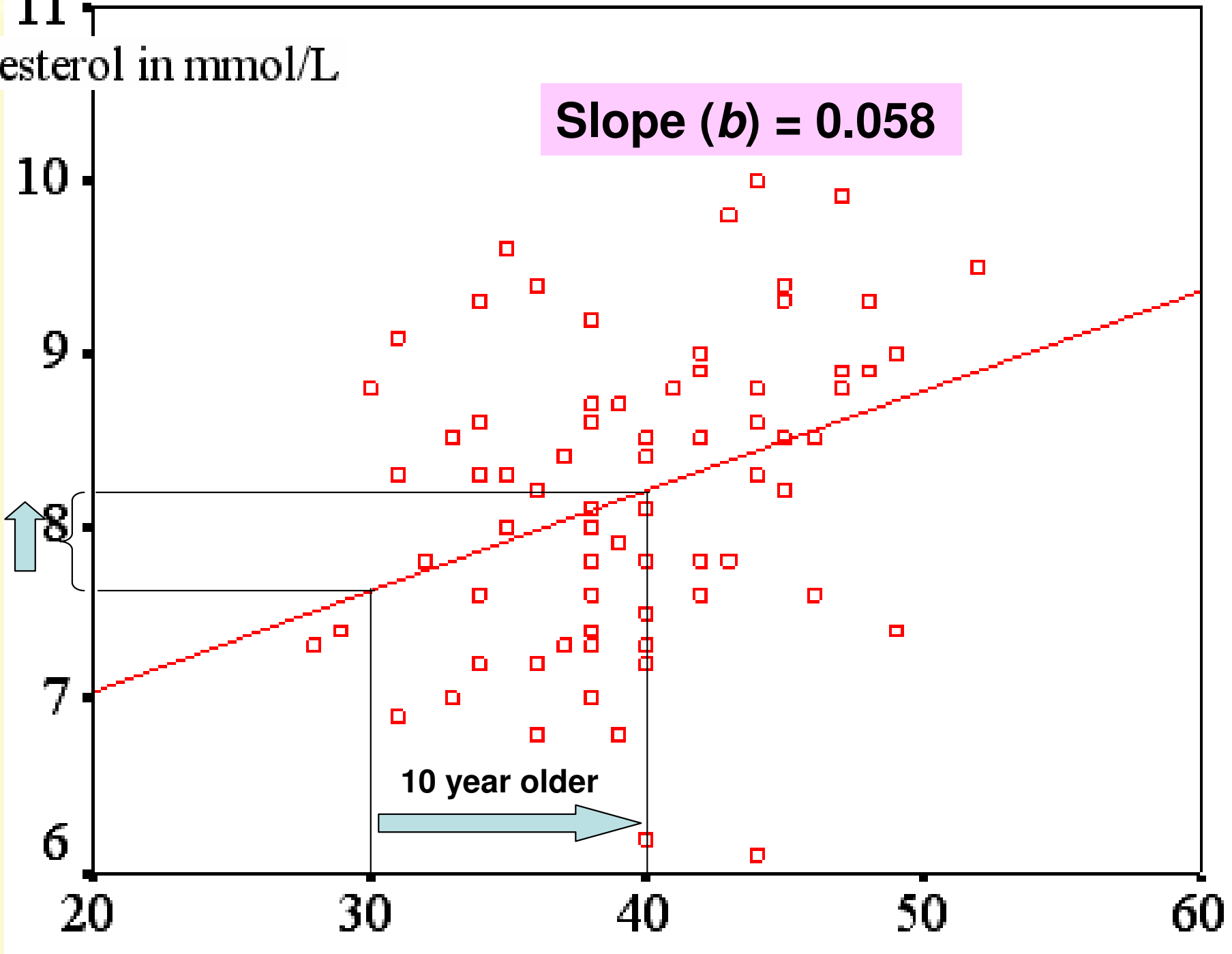
50

60

Age in year



Cholesterol in mmol/L

Slope (b) = 0.058~ 0.6
mmol/l

10 year older

20

30

40

50

60

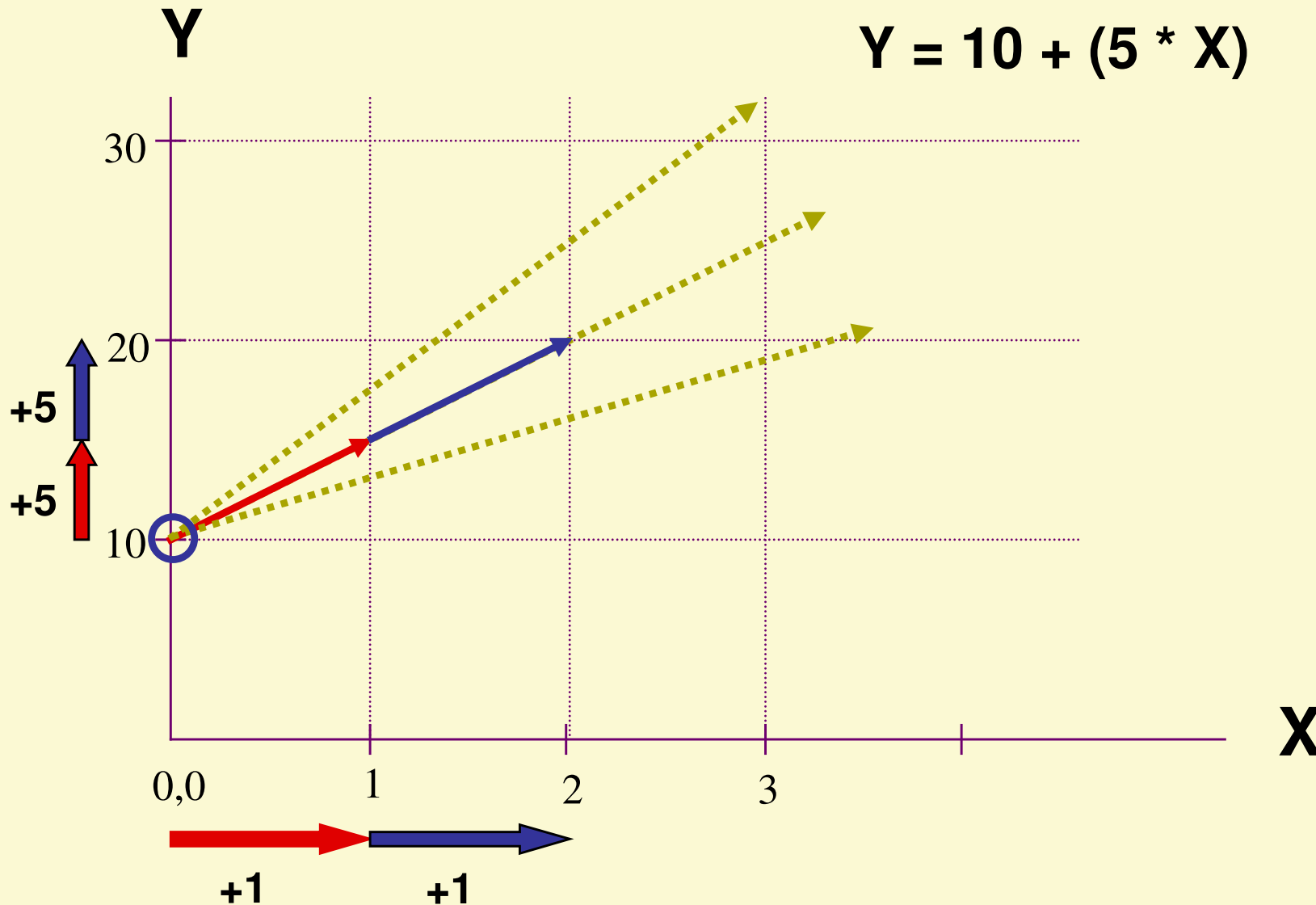
Age in year

The Linear line is described by the "Linear Equation".

$$Y = a + (b * X)$$

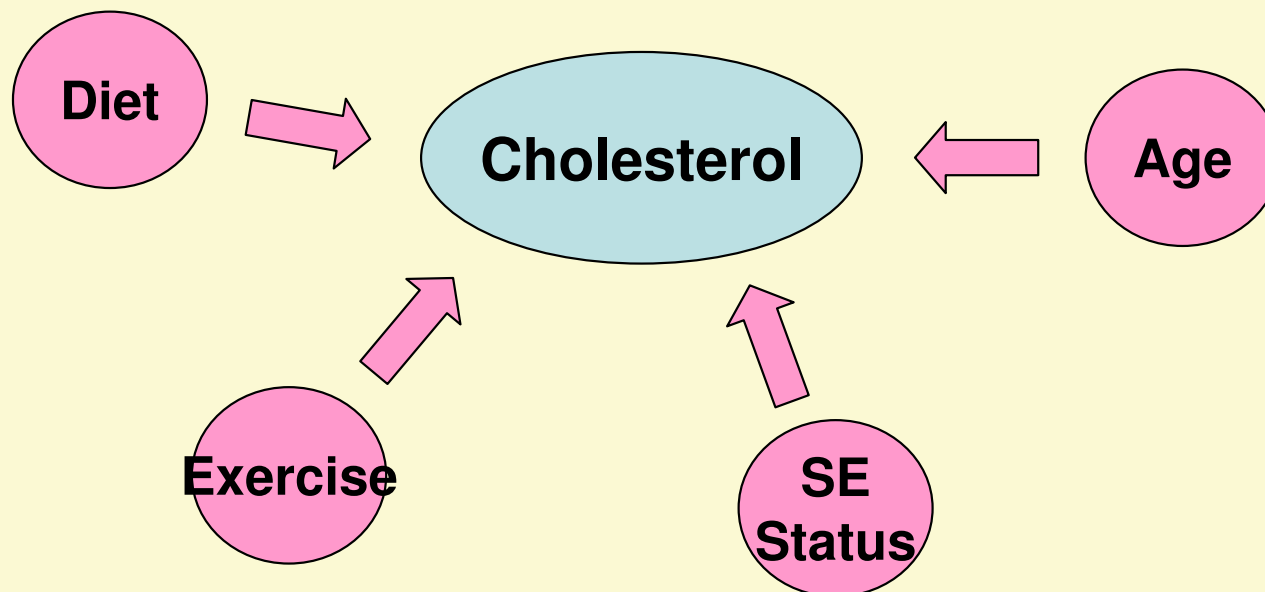
$$Y = \text{Constant} + (\text{slope} * X)$$

$$Y = 10 + (5 * X)$$

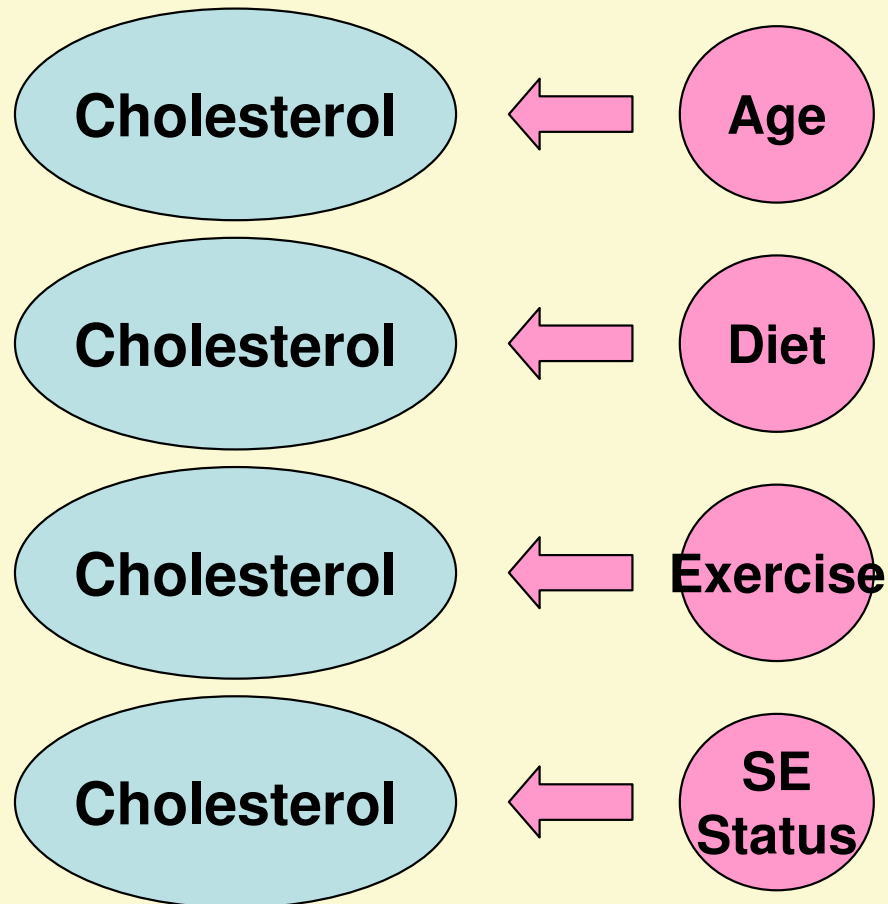


Basic Theory of MLR

- Most of the outcomes (events) are determined (influenced) by more than one factors (e.g. blood pressure, cholesterol level, etc.)

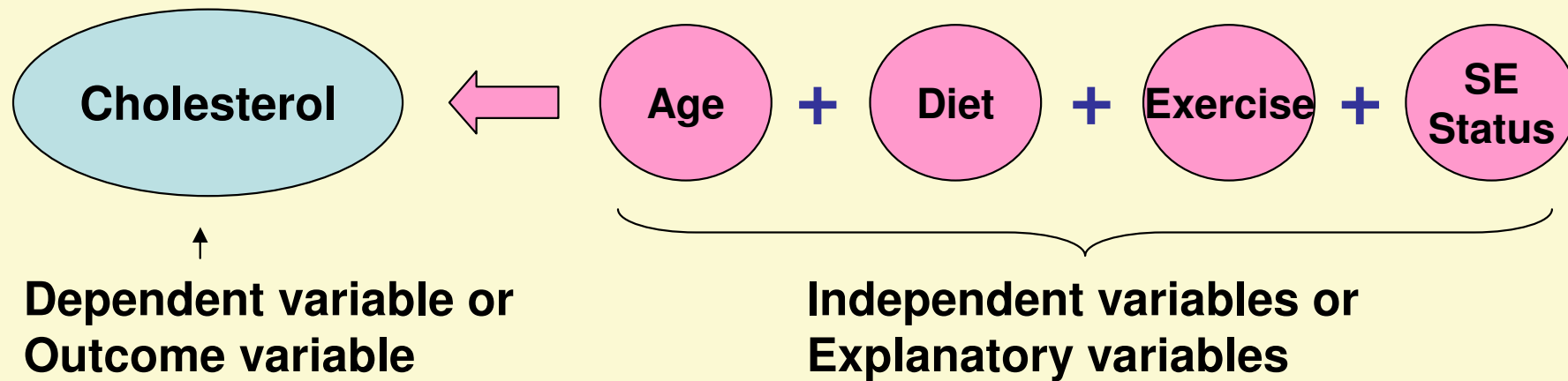


Basic Theory



- If we look at each factor to the outcome at one time, it will not be realistic.
- We should look at the relationship of these factors to the outcome at the same time.

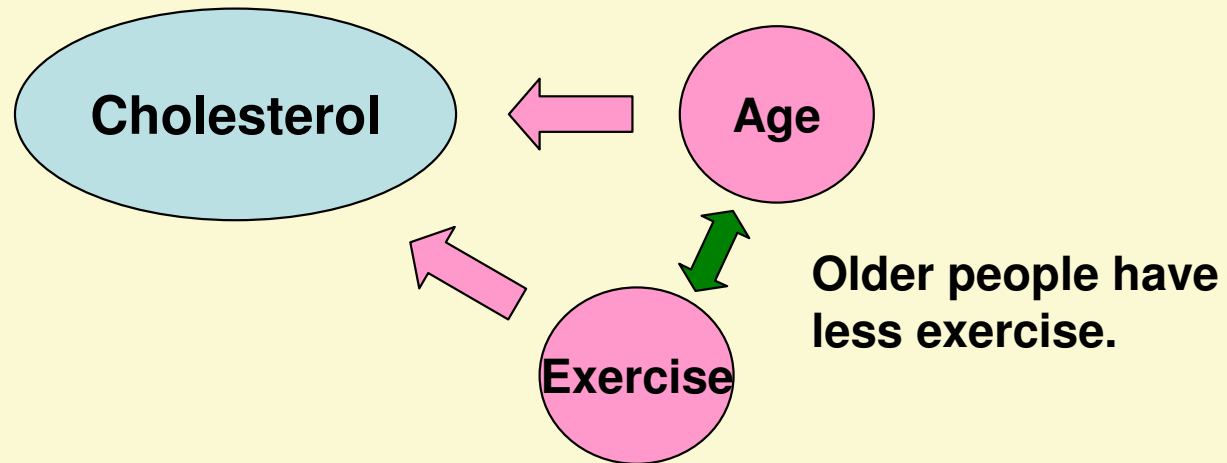
Basic Theory



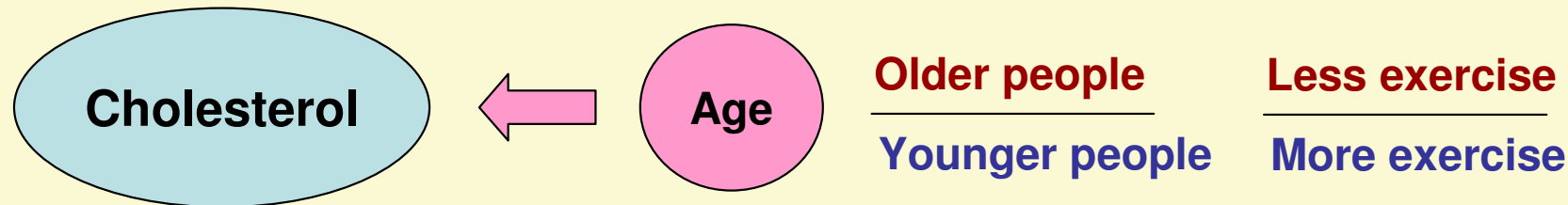
When we look at the relation of these factors (explanatory variables) to the outcome at the same time

- We will obtain the “independent effect” of explanatory variables to outcome.
- We can also study the “interaction” (IA) between independent variables (Synergistic/Antagonistic IA).

Independent Effect / Confounding



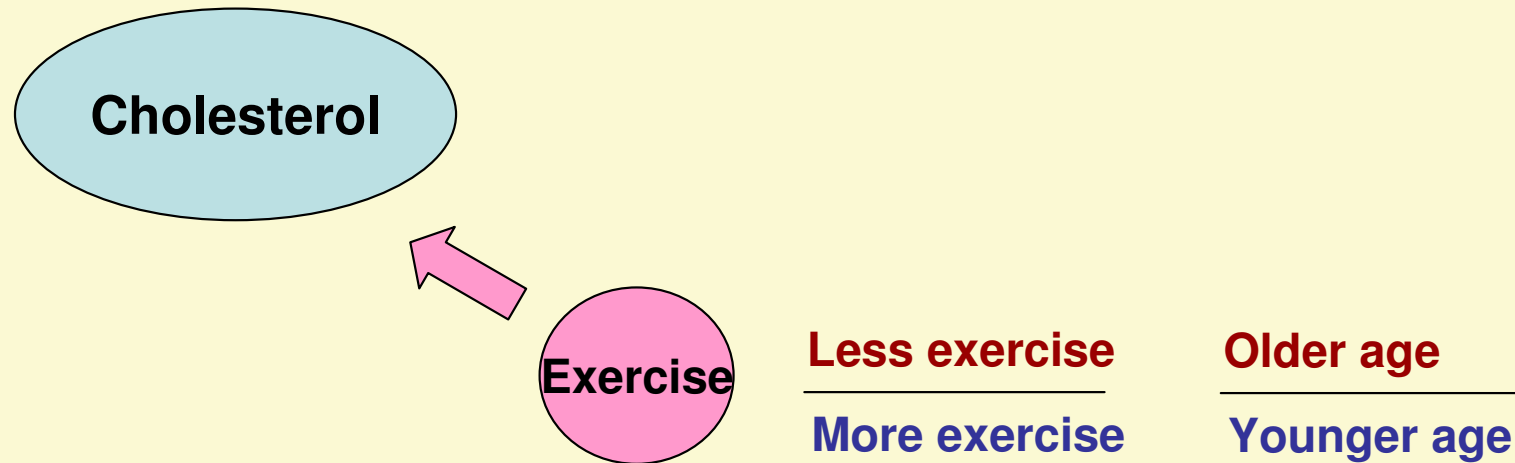
Independent Effect / Confounding



Effect that we found here, is not only the pure effect of age, but also additional effect from exercise. (Older people have less exercise – so that the relationship of being higher cholesterol among older age is exaggerated by the effect of less exercise).

In this example, the result (of the relationship between cholesterol and age) is confounded by exercise.

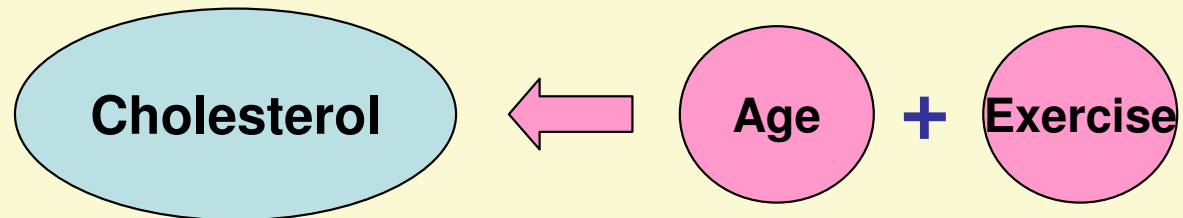
Independent Effect / Confounding



Effect that we found here, is not only the pure effect of exercise, but also additional effect from age. (Less exercise people are older people – so that the relationship of being higher cholesterol among less exercise people is exaggerated by the effect of older age).

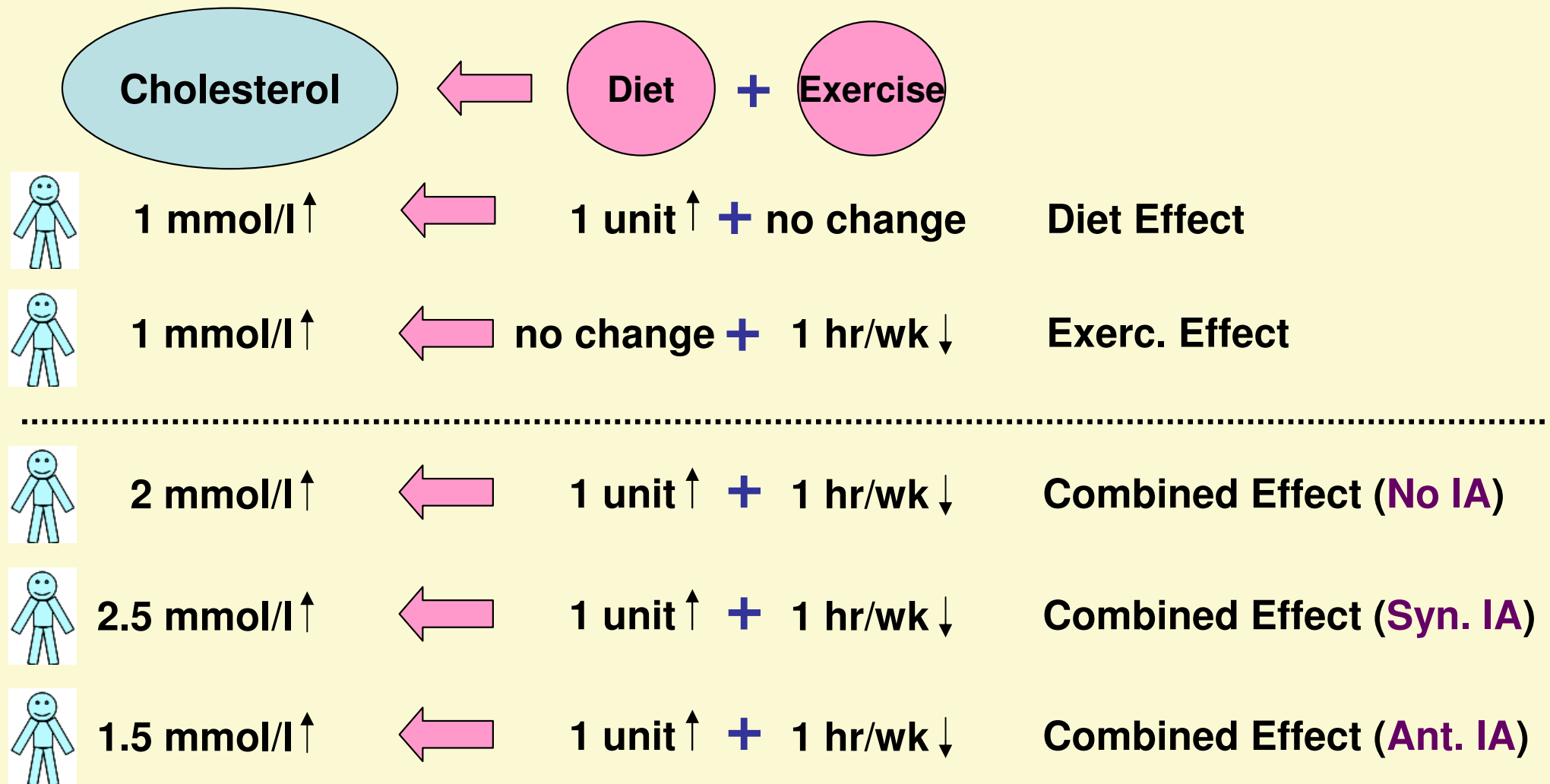
In this example, the result (of the relationship between cholesterol and exercise) is confounded by age.

Independent Effect / Confounding



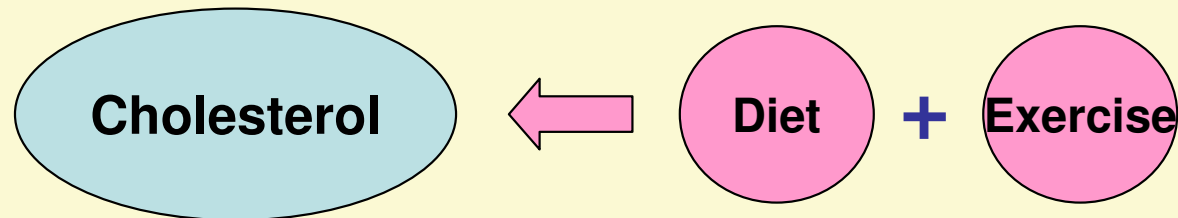
But, if we subject them together in the regression model, the confounding effects were eliminated and we can get the “independent effect” of each independent variable.

Interaction



IA=Interaction; Syn. IA=Synergistic Interaction; Ant. IA= Antagonistic Interaction

Interaction



Example:

Those with higher cholesterol diet, their cholesterol level will be higher.

Say, **1 unit more in cholesterol diet score**, cholesterol level will be higher for 1 mmol/L.

Those with less exercise, their cholesterol level will be higher.

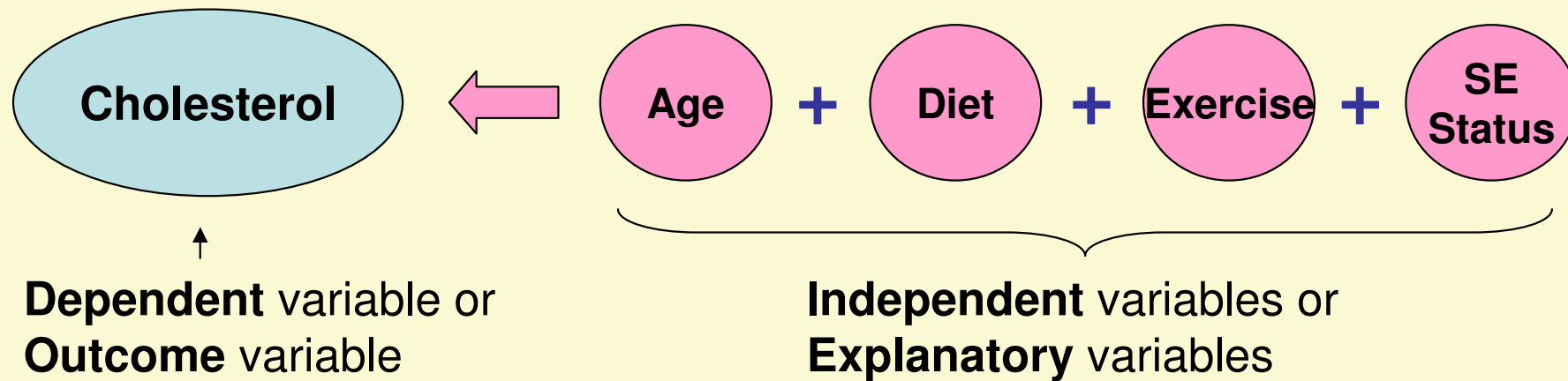
Say, **1 hour less exercise in a week**, cholesterol will higher for 1 mmol/L.

It means ... for **1 unit more in cholesterol diet AND 1 hour less exercise** in a week, there should be an increase in cholesterol for 2 mmol/L.

If it doesn't happen as above, but it increases for 3 mmol/L, it means that there is a **synergistic interaction** between diet and exercise.

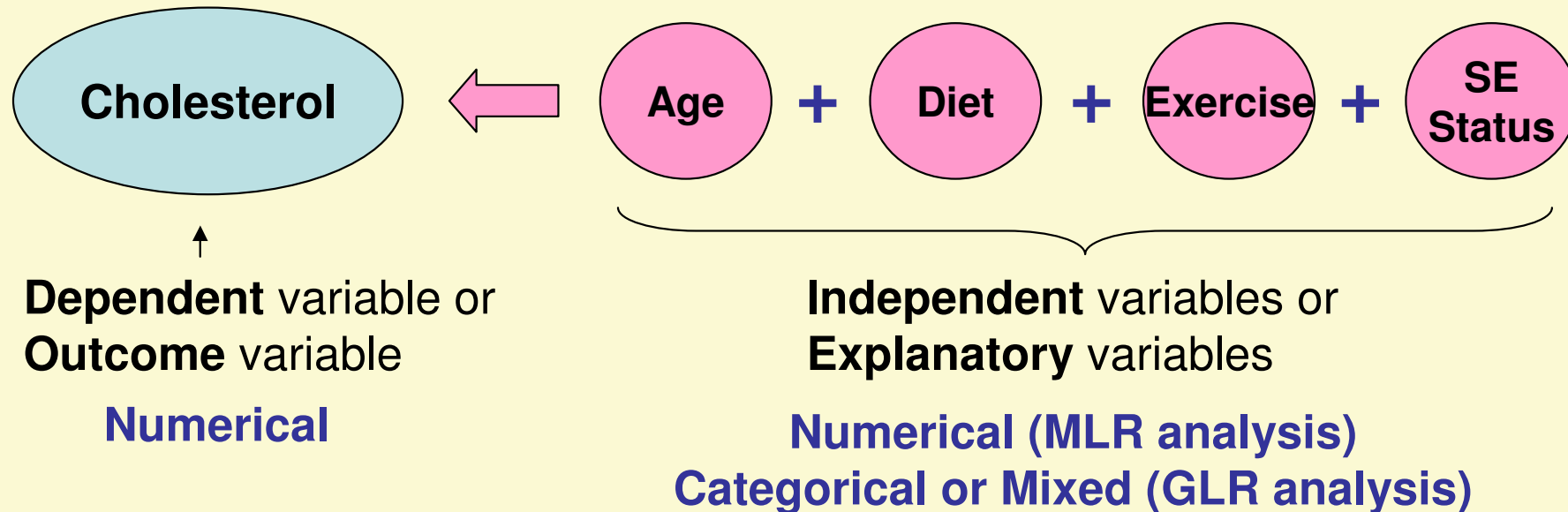
If it doesn't happen as above, but it increases only for 1.5 mmol/L, it means that there is an **antagonistic interaction** between diet and exercise.

Basic Theory



- **This analysis is used for**
 - **Exploring associated / influencing / risk factors to outcome (exploratory study)**
 - **Developing prediction model (exploratory study)**
 - **Confirming a specific relationship (confirmatory study)**

Basic Theory



$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \dots + \beta_n X_n$$

- If the dependent variable is numerical and independent variables are numerical, it will be called **Multiple Linear Regression** (MLR) analysis.
- MLR can be with categorical independent variables, but special name is given as **General Linear Regression** analysis.

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression

Step 3: Variable selection

⇒ **Preliminary main-effect model**

Step 4: Checking interaction & multicollinearity^a

⇒ **Preliminary final model**

Step 5: Checking model assumptions & outliers^a

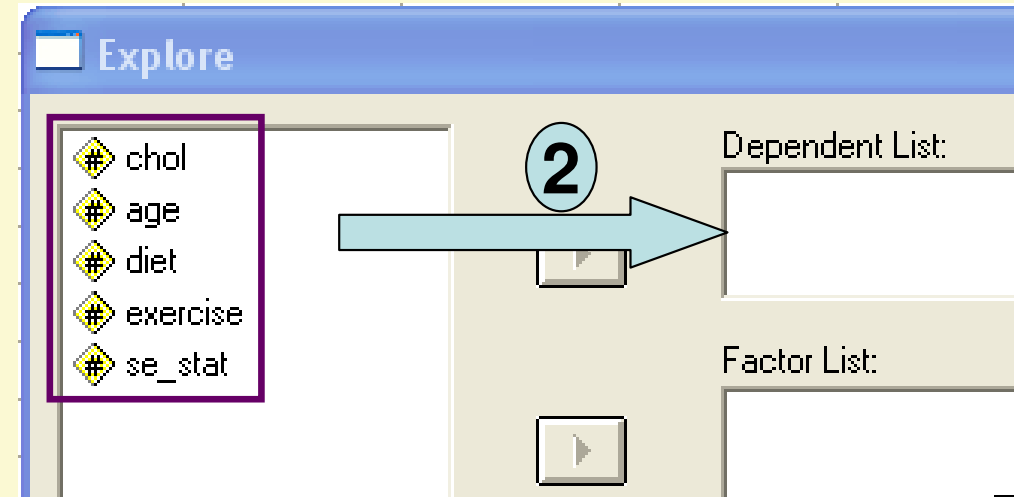
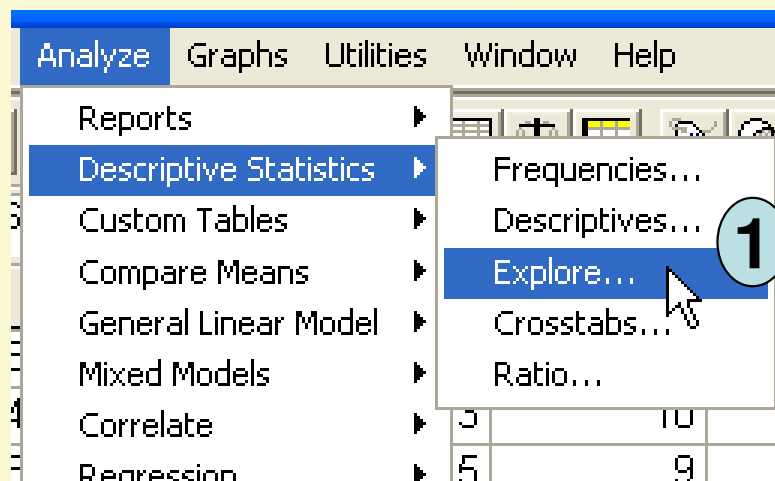
⇒ **Final model**

Step 6: Interpretation & data presentation

^a need remedial measures if problems are detected

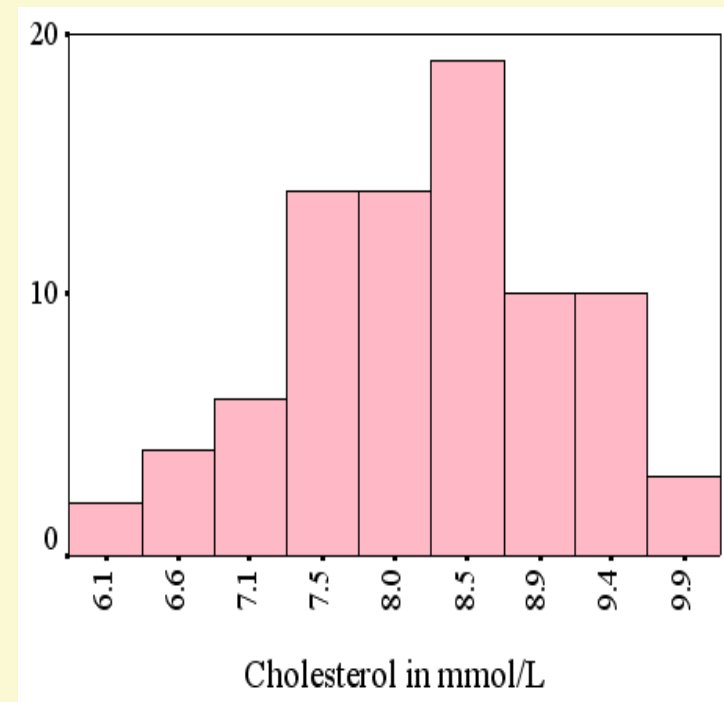
Neter J, Kutner MH, Nachtsheim CJ, Wasserman W. (1996). Applied linear statistical models (Fourth Ed.). Chicago: Irwin.

Step 1: Data Exploration



Descriptives

			Statistic
CHOL cholesterol in mmol/L	Mean		8.179
	95% Confidence Interval for Mean	Lower Bound	7.984
		Upper Bound	8.375
	5% Trimmed Mean		8.189
	Median		8.300
	Variance		.790
	Std. Deviation		.8890
	Minimum		6.1
	Maximum		10.0
	Range		3.9
	Interquartile Range		1.325
	Skewness		-.145
	Kurtosis		-.549



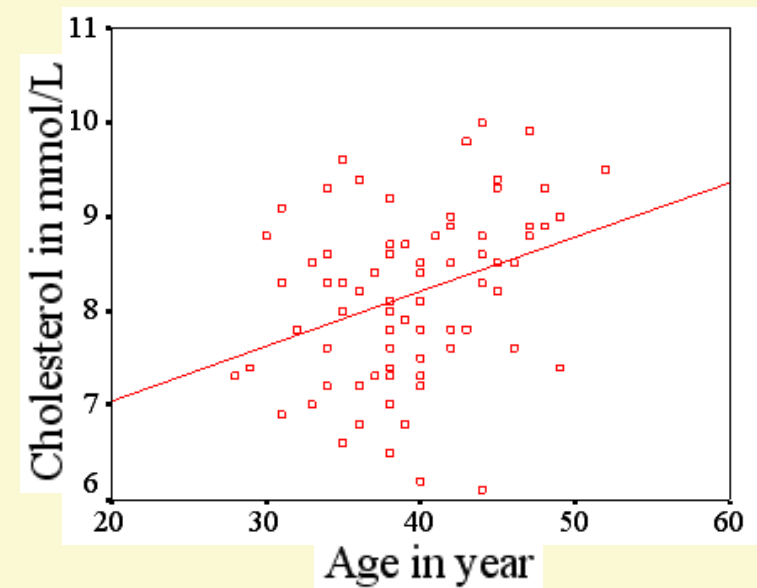
Step 2: Simple Linear Regression

Two main reasons:

- 1) To check the 'gross' relationship between dependent and each independent variable
- 2) Later this result will be compared with multiple linear regression result. This comparison indicates the confounding effects if it is present.

The screenshot illustrates the steps to create a simple scatterplot in SPSS:

1. The 'Graphs' menu is open, and the 'Scatter...' option is selected.
2. The 'Scatterplot' dialog box is shown, with the 'Define' button highlighted.
3. The 'Simple Scatterplot' sub-dialog is shown, with the 'chol' variable assigned to the Y Axis and the 'age' variable assigned to the X Axis.
4. The 'chol' variable is highlighted in the list of variables.



Step 2: Simple Linear Regression

The screenshot shows the SPSS interface with the following steps highlighted:

- 1**: Analyze > Regression > Linear...
- 2**: Dependent: chol
- 3**: Independent(s): age
- 4**: WLS >>
- 5**: Statistics... (Regression Coefficients: Estimates, Confidence intervals, Variance matrix)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	P value	95% Confidence Interval for B	
		B	Std. Error	Beta		Sig.	Lower Bound	Upper Bound
1	(Constant)	5.895	.735		8.026	.000	4.434	7.357
	AGE age in year	5.776E-02	.018	.331	3.134	.002	.021	.094

a. Dependent Variable: CHOL cholesterol in mmol

Slope (b) = 0.058 (95% CI: .021, .094)

Table 3: Factors associated with blood cholesterol level (mmol/L) among the study population ($n=82$) using simple linear regression

Independent Variable	SLR ^a	
	b (95%CI)	P value
Age (year)	0.06 (0.02, 0.09)	0.002
Duration of exercise (hrs/wk)	- 0.62 (- 0.79, - 0.46)	<0.001
Diet inventory score	0.45 (0.30, 0.61)	<0.001
Socio-economic index	0.21 (0.17, 0.25)	<0.001

^a Simple linear regression (Outcome as Cholesterol mmol/L)

b = crude regression coefficient

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression

Step 3: Variable selection

⇒ **Preliminary main-effect model**

Step 4: Checking interaction & multicollinearity^a

⇒ **Preliminary final model**

Step 5: Checking model assumptions & outliers^a

⇒ **Final model**

Step 6: Interpretation & data presentation

^a need remedial measures if problems are detected

Step 3: Variable Selection

- **Automatic / Manual methods**
 - Forward method
 - Backward method
 - Stepwise method
 - All possible models method
- **Nowadays, as computers are faster, automatic methods can be done easily.**
- **In SPSS, *forward*, *backward* and *stepwise* can be used.**
- **All 3 methods should be used for this step. Take the biggest model (all selected variables should be significant) for further analysis.**

Step 3: Variable Selection

The screenshot shows the SPSS Linear Regression dialog box. The 'Analyze' menu is open, and 'Linear...' is selected. The dialog box has the following settings:

- Dependent:** chol (indicated by arrow 2)
- Independent(s):** age, diet (indicated by arrow 3)
- Method:** Enter (indicated by arrow 4)
- Selection:** Enter, Stepwise, Remove, Backward (indicated by arrow 4)
- Case Labels:** (empty)
- Statistics...** button is highlighted (indicated by arrow 5)

The 'Linear Regression: Statistics' dialog box is also shown, with the following settings:

- Regression Coefficients:**
 - Estimates
 - Confidence intervals (indicated by arrow 6)
 - Covariance matrix
- Model fit:**
 - Model fit
 - R squared change
 - Descriptives
 - Part and partial correlations
 - Collinearity diagnostics
- Residuals:** (empty)

Result: Stepwise

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	5.845	.241		24.264	.000	5.366	6.325
	socio-economic status (index)	.211	.021	.748	10.069	.000	.170	.253
2	(Constant)	7.660	.587		13.048	.000	6.492	8.829
	socio-economic status (index)	.158	.025	.559	6.235	.000	.108	.208
	duration of exercise (hours/week)	-.288	.086	-.301	-3.352	.001	-.460	-.117
3	(Constant)	8.593	.633		13.574	.000	7.332	9.853
	socio-economic status (index)	1.369E-02	.052	.048	.262	.794	-.090	.118
	duration of exercise (hours/week)	-.550	.117	-.574	-4.688	.000	-.784	-.317
	diet inventory (higher the score, higher	.372	.120	.451	3.106	.003	.134	.610
4	(Constant)	7.297	.620		11.763	.000	6.062	8.532
	duration of exercise (hours/week)	-.540	.062	-.563	-8.702	.000	-.663	-.416
5	(Constant)	7.297	.620		11.763	.000	6.062	8.532
	duration of exercise (hours/week)	-.540	.062	-.563	-8.702	.000	-.663	-.416
	diet inventory (higher the score, higher cholesterol content)	.394	.052	.478	7.527	.000	.290	.498
	age in year	3.281E-02	.011	.188	2.914	.005	.010	.055

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \dots + \beta_n X_n$$

$$\text{Cholesterol} = 7.297 - (0.540 * \text{exercise}) + (0.394 * \text{diet}) + (0.033 * \text{age})$$

P values

Result: Forward

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	5.845	.241		24.264	.000	5.366	6.325
	socio-economic status (index)	.211	.021	.748	10.069	.000	.170	.253
2	(Constant)	7.660	.587		13.048	.000	6.492	8.829
	socio-economic status (index)	.158	.025	.559	6.235	.000	.108	.208
	duration of exercise (hours/week)	-.288	.086	-.301	-3.352	.001	-.460	-.117
3	(Constant)	8.593	.633		13.574	.000	7.332	9.853
	socio-economic status (index)	1.369E-02	.052	.048	.262	.794	-.090	.118
	duration of exercise (hours/week)	-.550	.117	-.574	-4.688	.000	-.784	-.317
	diet inventory (higher the score, higher cholesterol content)	.372	.120	.451	3.106	.003	.134	.610
4	(Constant)	7.151	.783		9.131	.000	5.591	8.710
	socio-economic status (index)	1.545E-02	.050	.055	.309	.758	-.084	.115
	duration of exercise (hours/week)	-.511	.113	-.533	-4.519	.000	-.736	-.286
	diet inventory (higher the score, higher cholesterol content)	.363	.114	.440	3.168	.002	.135	.591
	age in year	3.285E-02	.011	.188	2.900	.005	.010	.055

P values

a. Dependent Variable: cholesterol in mmol/L

Result: Backward

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	7.151	.783		9.131	.000	5.591	8.710
	age in year	3.285E-02	.011	.188	2.900	.005	.010	.055
	diet inventory (higher the score, higher cholesterol content)	.363	.114	.440	3.168	.002	.135	.591
	duration of exercise (hours/week)	-.511	.113	-.533	-4.519	.000	-.736	-.286
	socio-economic status (index)	1.545E-02	.050	.055	.309	.758	-.084	.115
2	(Constant)	7.297	.620		11.763	.000	6.062	8.532
	age in year	3.281E-02	.011	.188	2.914	.005	.010	.055
	diet inventory (higher the score, higher cholesterol content)	.394	.052	.478	7.527	.000	.290	.498
	duration of exercise (hours/week)	-.540	.062	-.563	-8.702	.000	-.663	-.416

P values

a. Dependent Variable: cholesterol in mmol/L

From the above 3 automatic procedures, we obtain the **preliminary main effect model** as:

$$\text{Cholesterol} = 7.297 - (0.540 \cdot \text{exercise}) + (0.394 \cdot \text{diet}) + (0.033 \cdot \text{age})$$

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression

Step 3: Variable selection

⇒ Preliminary main-effect model

Step 4: Checking interaction & multicollinearity^a

⇒ Preliminary final model

Step 5: Checking model assumptions & outliers^a

⇒ Final model

Step 6: Interpretation & data presentation

^a need remedial measures if problems are detected

Step 4.1: Checking Interactions

- All possible 2-ways interactions (ex*diet; ex*age; diet*age) are checked.
 - Interaction terms are calculated (Transform⇒Compute).
 - Add into the model as additional independent variable.
 - Run the model using ‘*enter*’.
 - If an interaction term is significant ($P < 0.05$), it means that there is an interaction between the 2 variables. And *therefore*, the appropriate model is the main effect variables plus the significant interaction term.
 - Check one interaction term at a time.
- In our example data, all 3 interaction terms are not significant. It means that no interaction term should be added.

Step 4.1: Checking Interactions

Compute Variable [X]

Target Variable: **age_diet** (2)

Numeric Expression: **age * diet** (3)

Transform Analyze (1)

Compute Variable..

Variables: chol, age, diet, exercise, se_stat

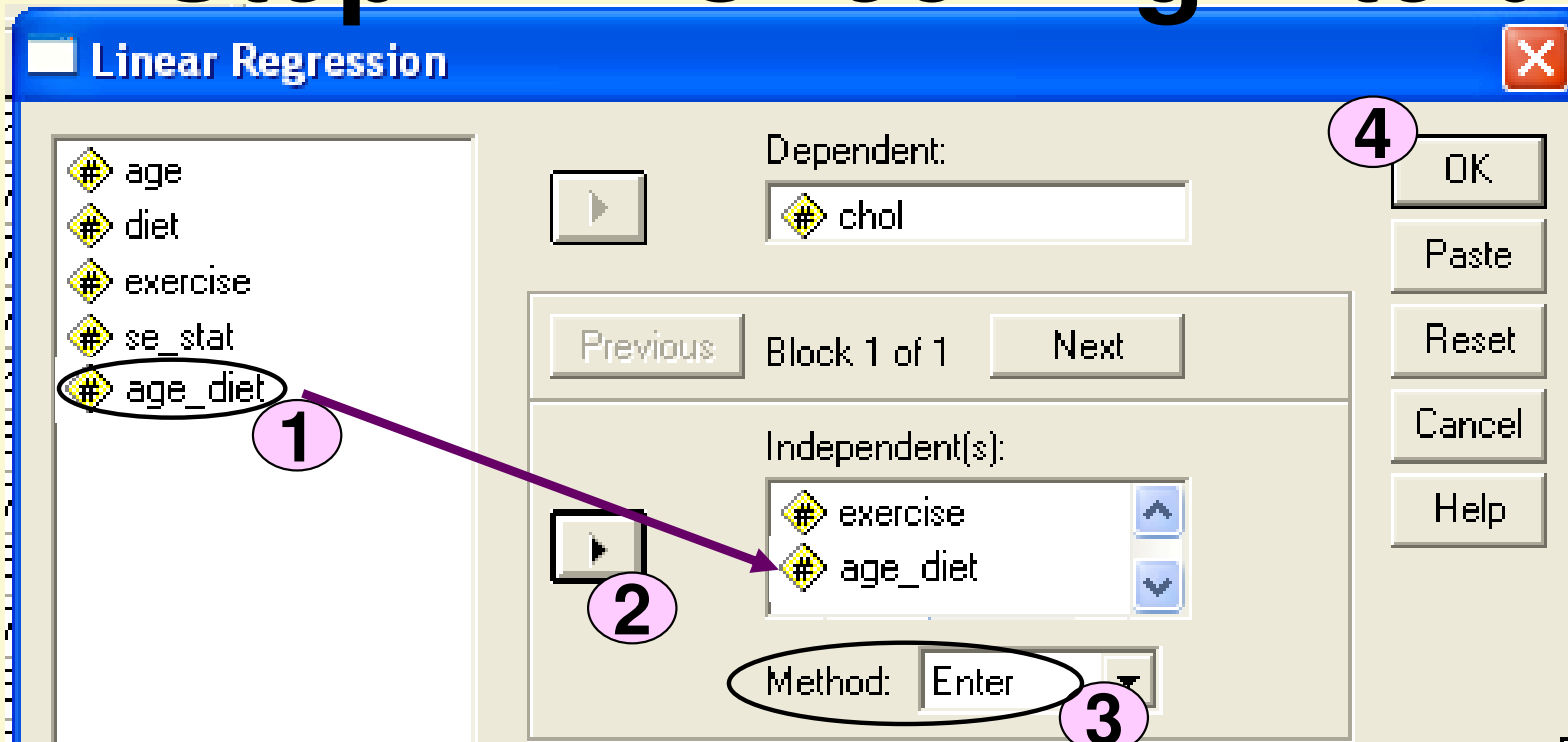
Functions: ABS(numexpr), ANY(test,value,value,...), ARSIN(numexpr), ARTAN(numexpr), CDFNORM(zvalue), CDF.BERNOULLI(q,p)

If... (3)

OK Paste Reset Cancel Help

	chol	age	diet	exercise	se_stat	age_diet
1	6.6	35	4	5	8	140.00
2	7.5	40	3	3	10	120.00
3	7.9	39	5	5	9	195.00
4	7.4	38	3	4	7	114.00
5	6.9	31	3	4	9	93.00

Step 4.1: Checking Interactions

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Lower
		B	Std. Error	Beta			
1	(Constant)	6.812	2.102		3.240	.002	
	age in year	4.490E-02	.051	.257	.874	.385	
	diet inventory (higher the score, higher cholesterol content)	.495	.421	.600	1.176	.243	
	duration of exercise (hours/week)	-.539	.063	-.562	-8.610	.000	
	AGE_DIET	2.53E-03	.010	-.144	-.241	.810	

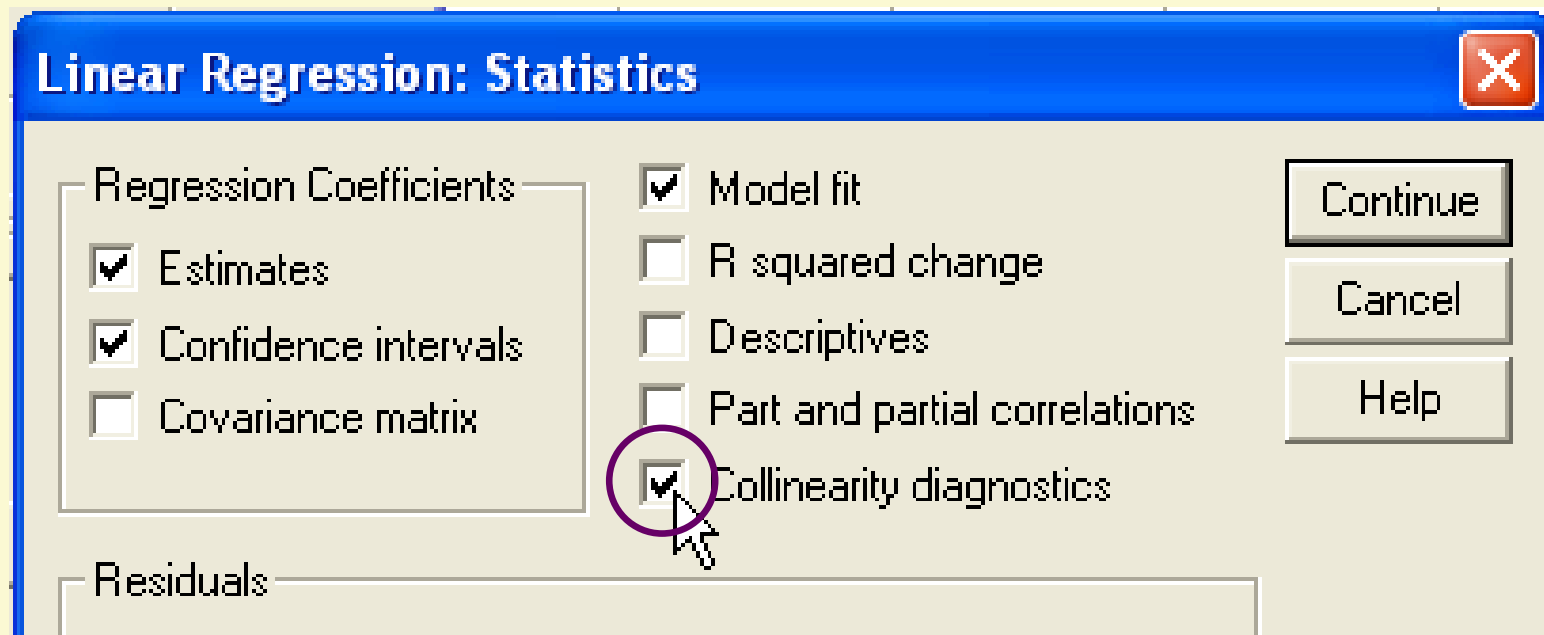
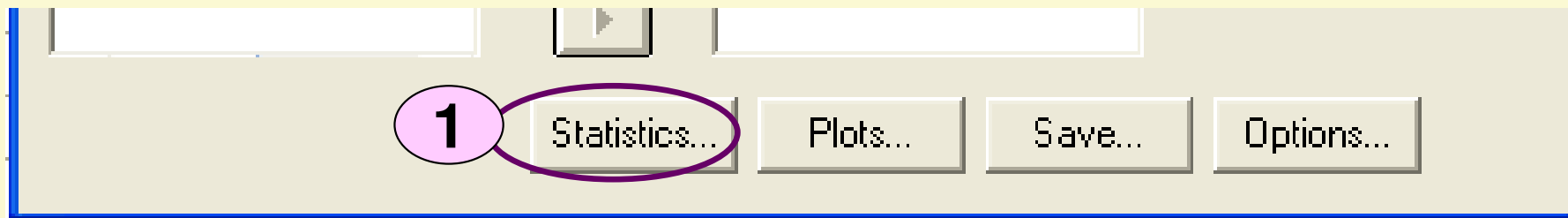
a. Dependent Variable: cholesterol in mmol/L

Step 4.2: Checking Multicollinearity (MC)

- If the independent variables are highly correlated, the regression model is said to be “statistically not stable”.
 - P values of the involved variables are considerably larger (than what it should be).
 - The width of 95% CI of the regression coefficients are larger.
 - Appropriate variables may be rejected wrongly.
 - Therefore, statistically, it is said that ‘the model is not stable’.
- We have to check the obtained model whether this kind of problem (MC) exists or not.

Step 4.2: Checking Multicollinearity (MC)

- Just run the Preliminary main effect model by using 'enter', and click 'collinearity diagnostic' in 'statistics'.



Step 4.2: Checking Multicollinearity (MC)

- Just run the Preliminary main effect model by using 'enter', and click 'collinearity diagnostic' in 'statistics'.

Model		Unstandardized Coefficients		95% Confidence Interval for B		Collinearity Statistics	
		B	Std. Error	Lower Bound	Upper Bound	Tolerance	VIF
1	(Constant)	7.297	.70	6.062	8.532		
	age in year	3.281E-02	.15	.010	.055	.957	1.045
	diet inventory (higher the score, higher cholesterol content)	.394	.10	.290	.498	.988	1.012
	duration of exercise (hours/week)	-.540	.10	-.663	-.416	.950	1.053

a. Dependent Variable: cholesterol in mmol/L

Look at VIF (Variance-inflation factor). VIF measures the extent of multicollinearity problem. If VIF is more than 10, the problem needs remedial measures. Consult a statistician.

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression

Step 3: Variable selection

⇒ Preliminary main-effect model

Step 4: Checking interaction & multicollinearity^a

⇒ Preliminary final model

Step 5: Checking model assumptions & outliers^a

⇒ Final model

Step 6: Interpretation & data presentation

^a need remedial measures if problems are detected

Step 5: Checking model assumptions

- Assumptions are ...

- Random sample*

- Linearity

- Independence*

- Normality

- Equal variance

Overall linearity / Model fitness

Linearity of each indep. variable

LINE

* It is related to the study design.

- All are performed by using **residual plots**.

- A **residual** means “observed value” minus “predicted value” of dependent variable.

Step 5: Checking model assumptions

Steps to calculate residuals ...

The image shows two overlapping SPSS dialog boxes for Linear Regression. The left box is the main 'Linear Regression' dialog, and the right box is the 'Linear Regression: Save' sub-dialog. Four red circles with numbers 1, 2, 3, and 4 highlight specific steps:

- 1**: The 'Save...' button at the bottom of the 'Linear Regression' dialog.
- 2**: The 'Unstandardized' checkbox under the 'Predicted Values' section of the 'Linear Regression: Save' dialog.
- 3**: The 'Unstandardized' checkbox under the 'Residuals' section of the 'Linear Regression: Save' dialog.
- 4**: The 'Continue' button at the bottom right of the 'Linear Regression: Save' dialog.

Step 5: Checking model assumptions

	age	diet	exercise	chol	pre 1	res 1
1	35	4	5	6.6	7.32127	-.72127
2	40	3	3	7.5	8.17117	-.67117
3	39	5	5	7.9	7.84649	.05351
4	38	3	4	7.4	7.56563	-.16563
5	31	3	4	6.9	7.33597	-.43597
6	31	5	4	8.3	8.12397	.17603
7	38	6	5	7.6	8.20768	-.60768
8	48	4	3	8.9	8.82763	.07237
9	39	5	5	7.9	7.84649	.05351
10	38	7	5	8.6	8.60168	-.00168

$$\text{Chol (pred.)} = 7.297 - (0.540 * \text{exercise}) + (0.394 * \text{diet}) + (0.033 * \text{age})$$

$$\text{Chol (pred.)} = 7.297 - (0.540 * 5) + (0.394 * 4) + (0.033 * 35)$$

$$\text{Chol (pred.)} = 7.32$$

$$\text{Residual} = \text{Chol (observed)} - \text{Chol (pred.)} = 6.6 - 7.32 = -0.72$$

Step 5: Checking model assumptions

	age	diet	exercise	Data chol	Statistical Model pre_1	Discrepancy res_1
1	35	4	5	6.6	7.32127	-.72127
2	40	3	3	7.5	8.17117	-.67117
3	39	5	5	7.9	7.84649	.05351
4	38	3	4	7.4	7.56563	-.16563
5	31	3	4	6.9	7.33597	-.43597
6	31	5	4	8.3	8.12397	.17603
7	38	6	5	7.6	8.20768	-.60768
8	48	4	3	8.9	8.82763	.07237
9	39	5	5	7.9	7.84649	.05351
10	38	7	5	8.6	8.60168	-.00168

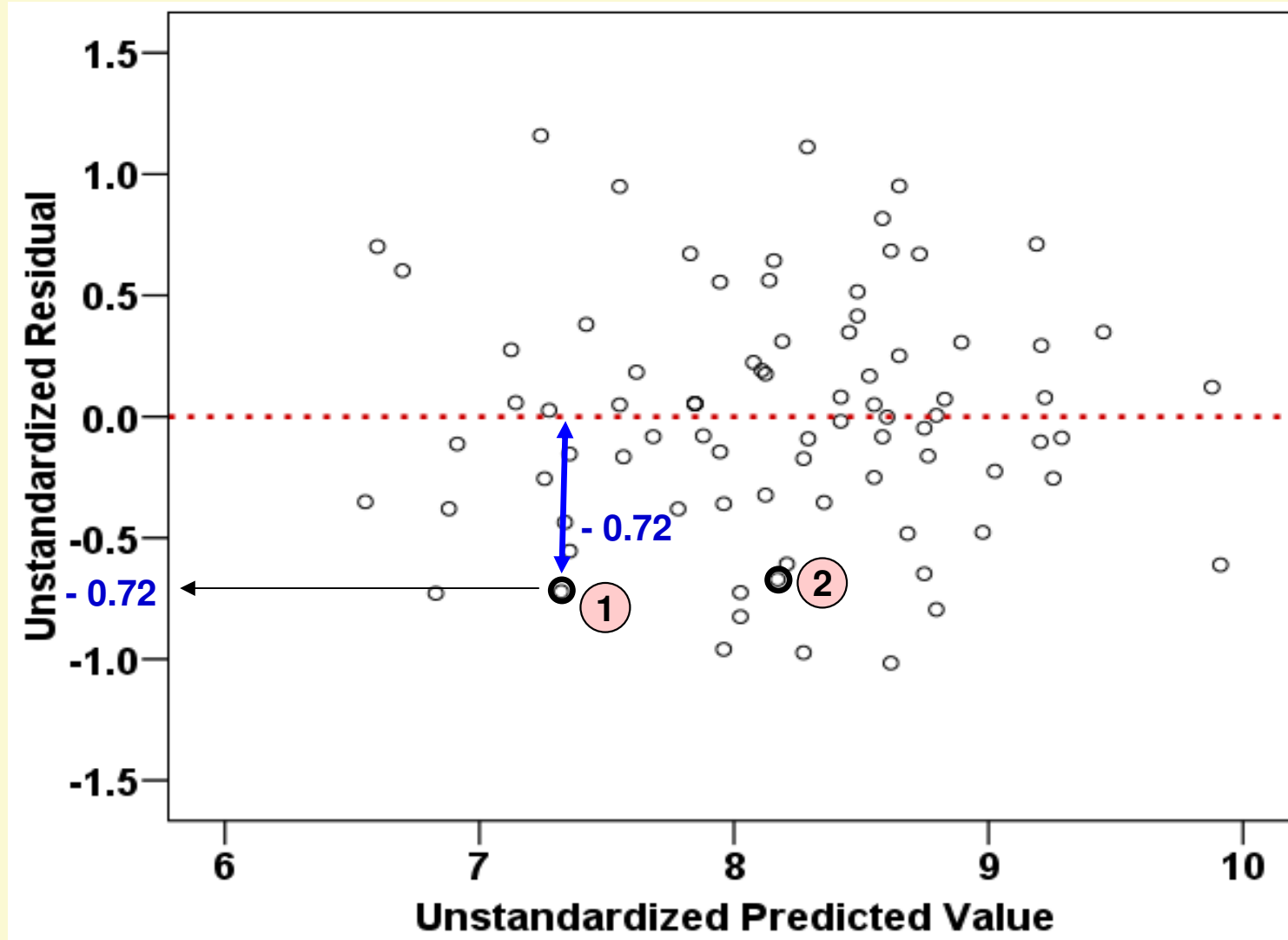
$$\text{Chol (pred.)} = 7.297 - (0.540 \cdot \text{exercise}) + (0.394 \cdot \text{diet}) + (0.033 \cdot \text{age})$$

$$\text{Chol (pred.)} = 7.297 - (0.540 \cdot 5) + (0.394 \cdot 4) + (0.033 \cdot 35)$$

$$\text{Chol (pred.)} = 7.32$$

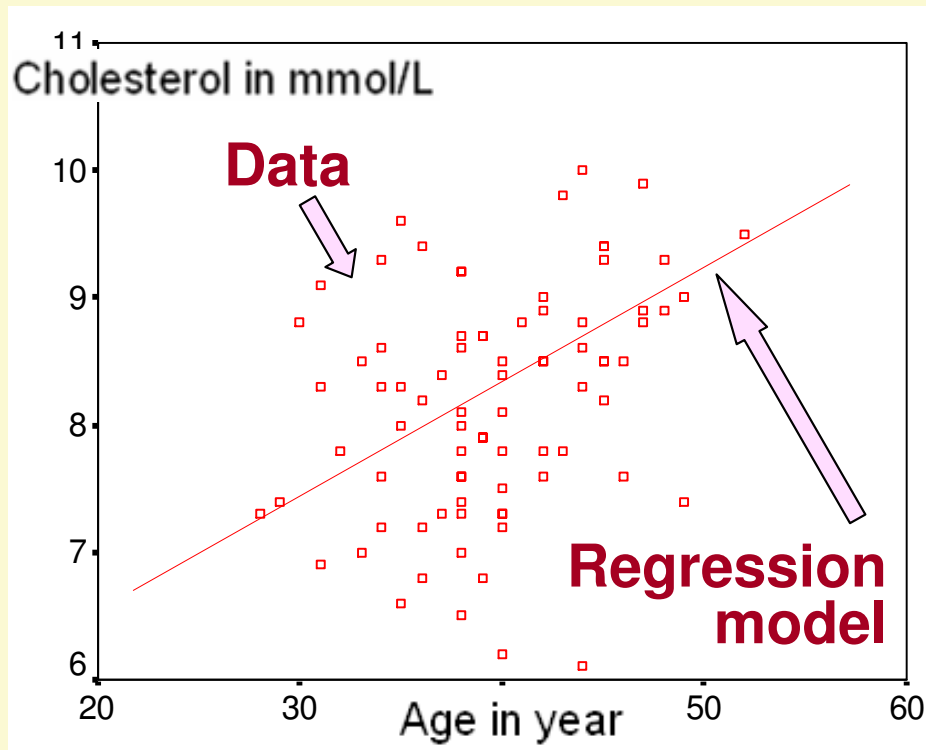
$$\text{Residual} = \text{Chol (observed)} - \text{Chol (pred.)} = 6.6 - 7.32 = -0.72$$

Step 5: Checking model assumptions

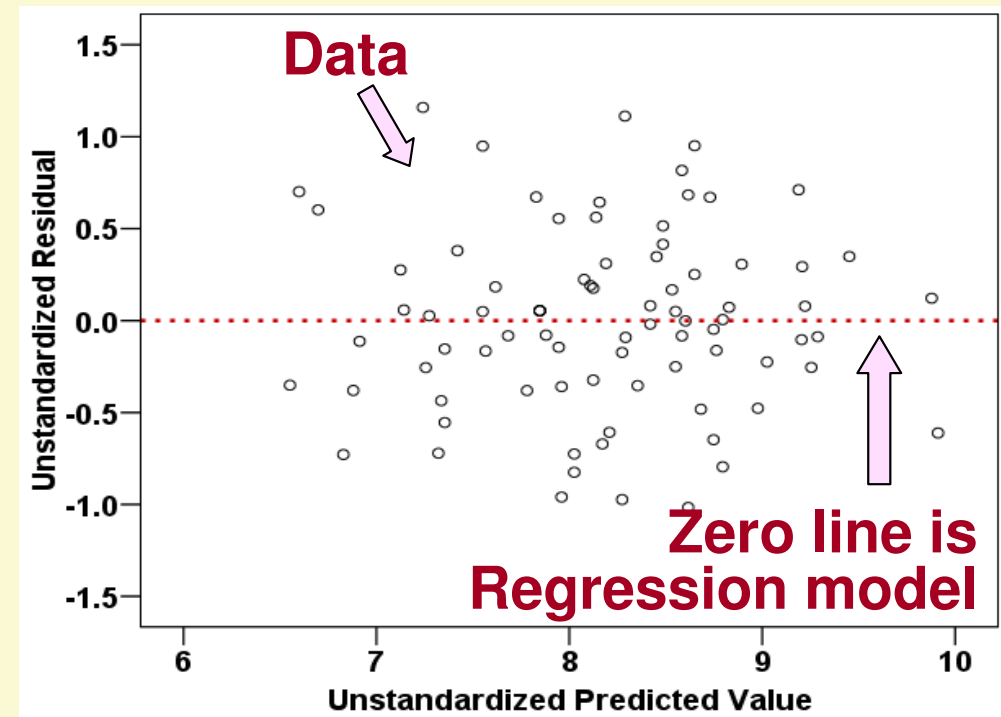


	age	diet	exercise	chol	pre_1	res_1
1	35	4	5	6.6	7.32127	-.72127
2	40	3	3	7.5	8.17117	-.67117

Step 5: Checking model assumptions



Simple Linear Regression



Multiple Linear Regression

Step 5: Checking model assumptions

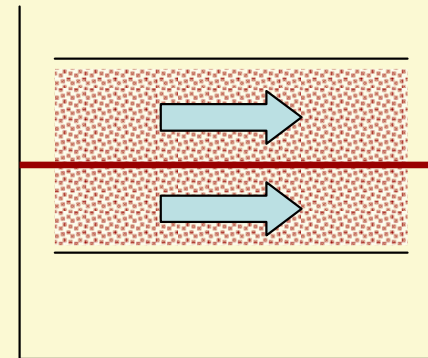
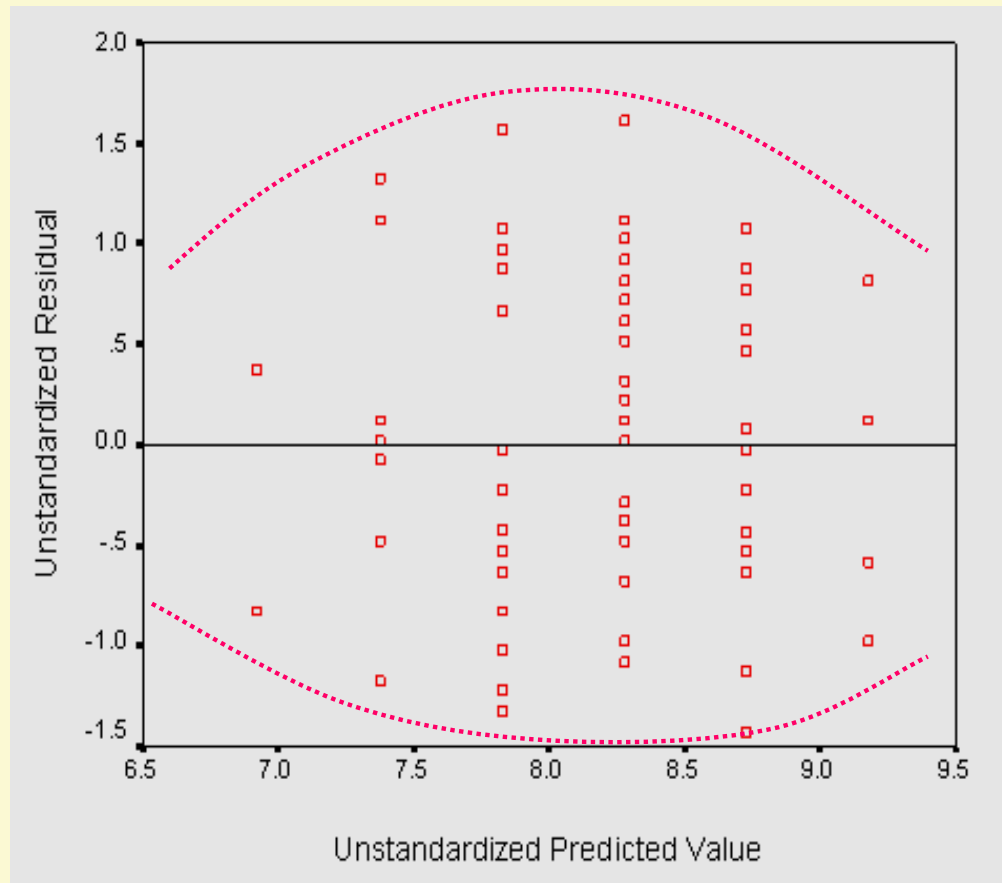
- Assumptions are ...

- Random sample*
 - **Linearity**
 - **Independence***
 - **Normality**
 - **Equal variance**
- LINE** {
- Overall linearity / Model fitness
- Linearity of each indep. variable
- * It is related to the study design.

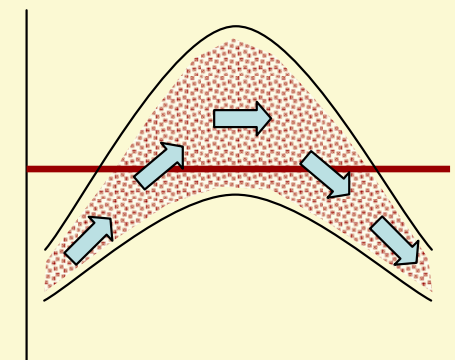
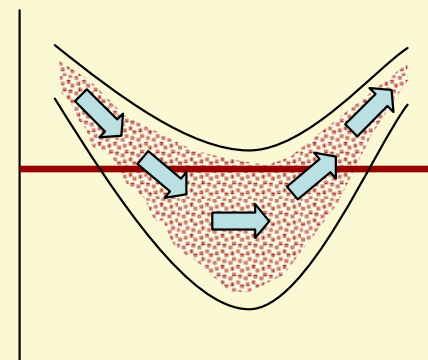
	3 types of residual plot	Assumptions
1.	Scatter plot: Residuals vs Predicted	Linearity – overall fitness Equal variance of residuals
2.	Histogram of residuals	Normality of residuals
3.	Scatter plot: Residuals vs each indep. var. (numerical)	Linearity of each indep. Var. numerical

Step 5: Checking model assumptions

OVERALL LINEARITY

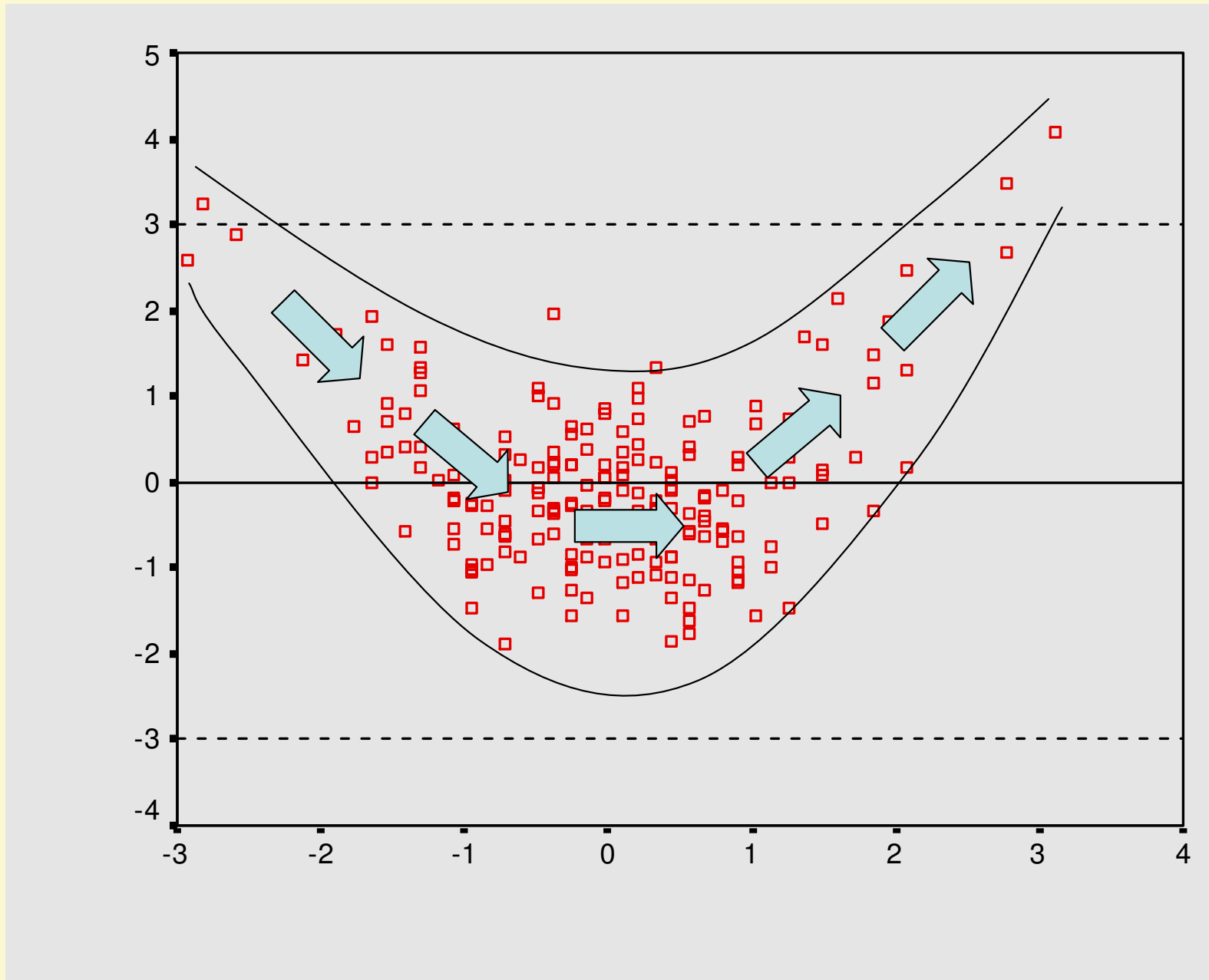


Linearity assumption is met (linear model fits well).

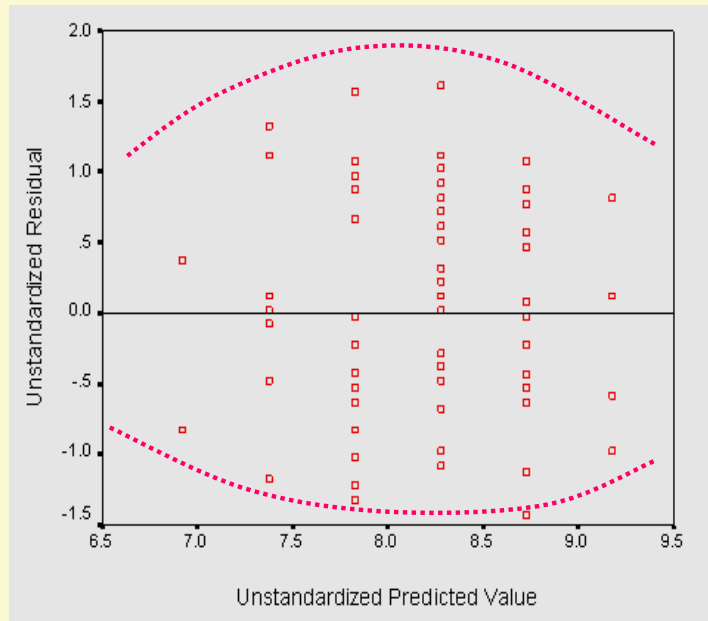


Linear assumption is not met (linear model doesn't fit well).

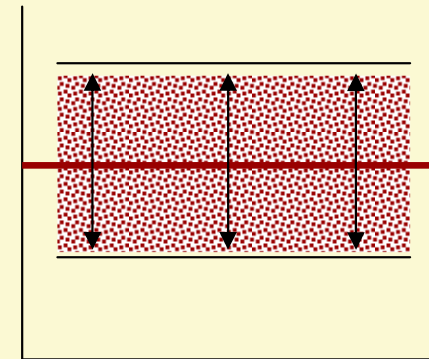
An example of non-linear relationship



Step 5: Checking model assumptions

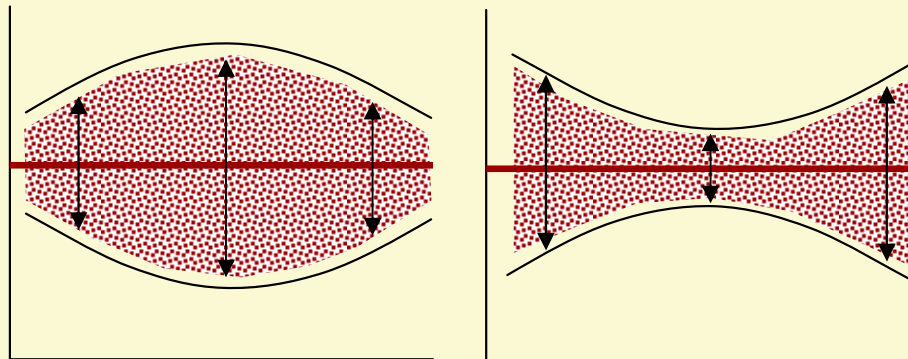


EQUAL VARIANCE

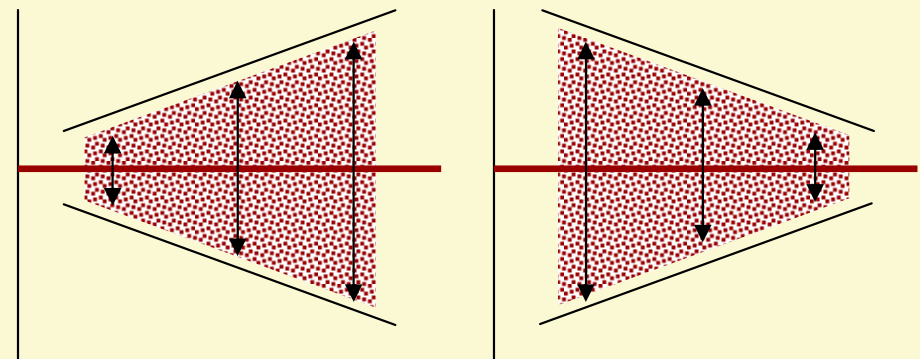


Equal-
variance
assumption is
met.

"Constantly increasing or decreasing"



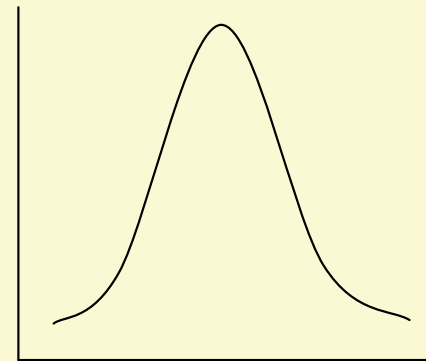
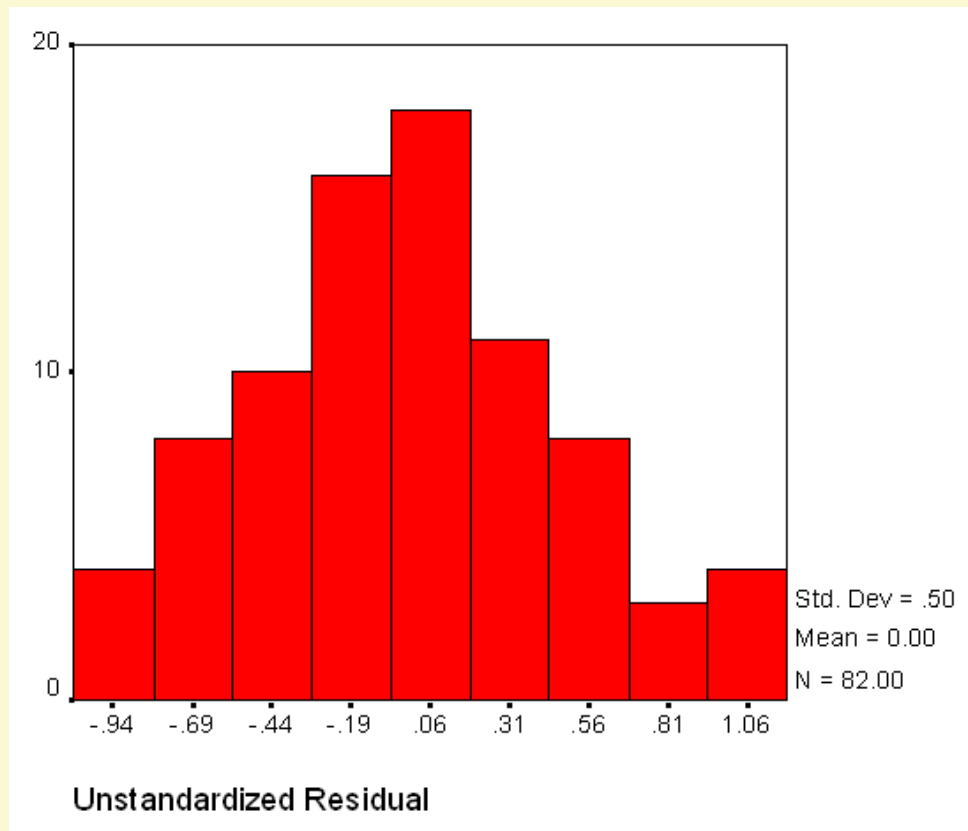
Equal-variance assumption is met.



Equal-variance assumption is **NOT** met.

Step 5: Checking model assumptions

NORMALITY

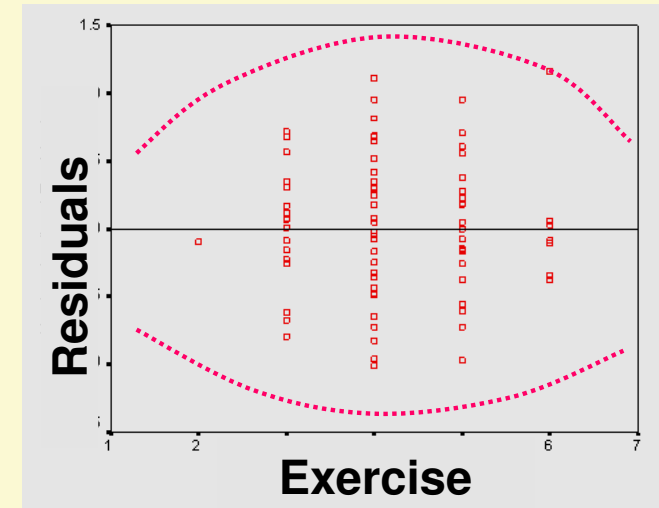
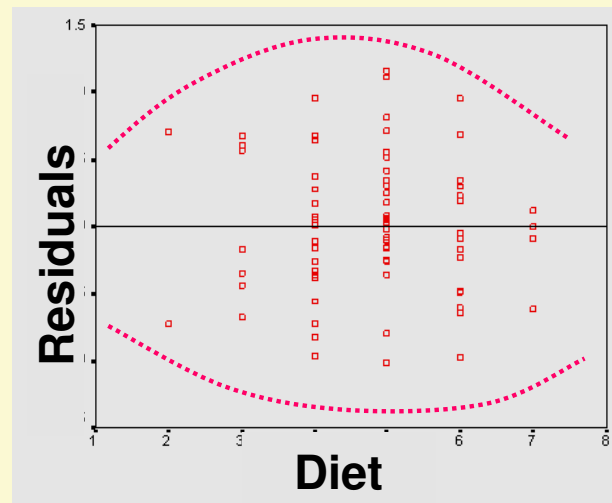
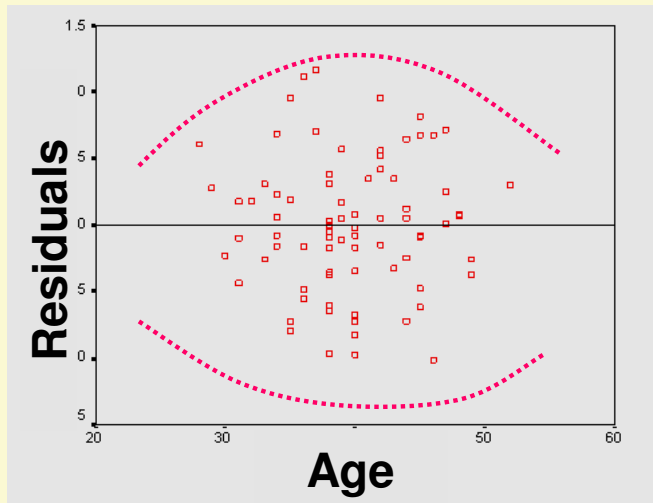


**Normality
assumption is
met.**



Normality assumption is not met.

Checking linearity of each numerical independent variables



- If there is no relationship between residuals and a numerical independent variable, the relationship of the independent variable with the outcome is linear.
- In above example, all are considered linear relationship.
- If not linear, we may need to transform data (*see statistician*).

Checking Outliers

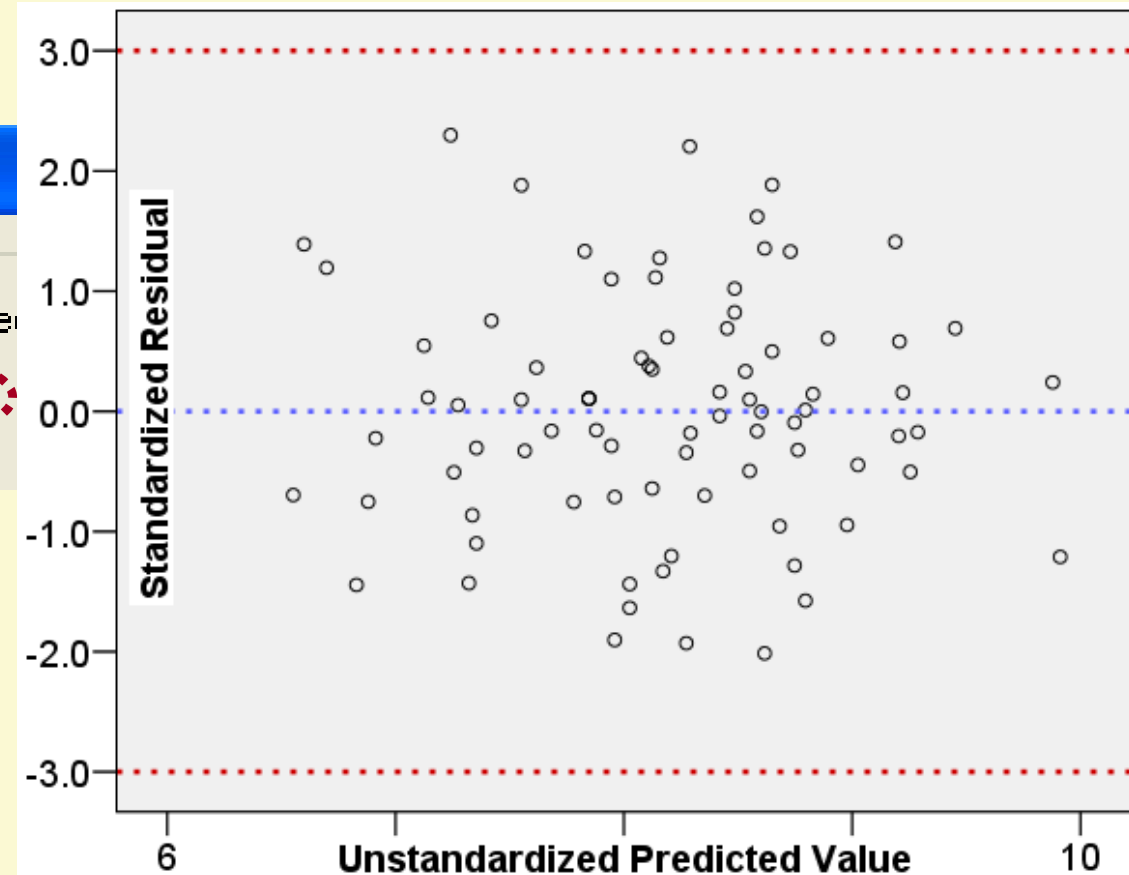
Linear Regression: Save

Predicted Values

- Unstandardized
- Standardized
- Adjusted

Residuals

- Unstandardized
- Standardized
- Studentized



- If data points are beyond +3 and -3 of standardized residuals, they are considered “outliers”.
- Check for ‘data entry error’ and ‘eligibility’ as study subjects. If no entry error and are an eligible cases, consult a statistician to handle the outliers.

Steps in Handling MLR

Step 1: Data exploration (Descriptive Statistics)

Step 2: Scatter plots and Simple Linear Regression

Step 3: Variable selection

⇒ Preliminary main-effect model

Step 4: Checking interaction & multicollinearity^a

⇒ Preliminary final model

Step 5: Checking model assumptions & outliers^a

⇒ Final model

Step 6: Interpretation & data presentation

^a need remedial measures if problems are detected

Step 6: Presentation/Interpretation

Table 4: Factors associated with blood cholesterol level (mmol/L) among the study population (n=82)

Variables	SLR ^a		MLR ^b		
	<i>b</i> ^c (95% CI)	<i>P</i> value	<i>Adj.b</i> ^d (95%CI)	<i>t</i> -stat.	<i>P</i> value
Age (year)	0.06 (0.02, 0.09)	0.002	0.03 (0.01, 0.06)	2.91	0.005
Duration of exercise (hrs/wk)	-0.62 (-0.79, -0.46)	<0.001	-0.54 (-0.66, -0.42)	- 8.70	<0.001
Diet inventory score	0.45 (0.30, 0.61)	<0.001	0.39 (0.29, 0.50)	7.53	<0.001
Socio-economic index	0.21 (0.17, 0.25)	<0.001	-	-	-

^a Simple linear regression

^b Multiple linear regression ($R^2=0.69$; The model reasonably fits well; Model assumptions are met; There is no interaction between independent variables, and no multicollinearity problem)

^c Crude regression coefficient

^d Adjusted regression coefficient

- For prediction study, it is essential to report the final model (equation).

$$\text{Chol (pred.)} = 7.30 + (0.03 * \text{age}) - (0.54 * \text{exercise}) + (0.39 * \text{diet})$$

Step 6: Presentation/Interpretation

Table 4: Factors associated with blood cholesterol level (mmol/L) among the study population ($n=82$)

Variables	SLR ^a		MLR ^b		
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Socio-economic index	0.21 (0.17, 0.25)	<0.001	-	-	-

- There is a significant linear relationship between age and cholesterol level ($P=0.005$). Those with 10 years older have cholesterol level higher for 0.3 mmol/L (95% CI: 0.1, 0.6 mmol/L).
- There is a significant linear relationship between duration of exercise and cholesterol level ($P<0.001$). Those having 1 hr/wk less exercise have cholesterol level higher for 0.54 mmol/L (95% CI: 0.66, 0.42 mmol/L).

Step 6: Presentation/Interpretation

Table 4: Factors associated with blood cholesterol level (mmol/L) among the study population ($n=82$)

Variables	SLR ^a		MLR ^b		
	b^c (95% CI)	P value	Adj. b^d (95%CI)	t -stat.	P value
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Diet inventory score	0.45 (0.30, 0.61)	<0.001	0.39 (0.29, 0.50)	7.53	<0.001
Socio-economic index	0.21 (0.17, 0.25)	<0.001	-	-	-

- There is a significant linear relationship between diet inventory index and cholesterol level ($P<0.001$). Those with 1 unit more in the index, have cholesterol level higher for 0.39 mmol/L (95% CI: 0.29, 0.50 mmol/L).
- With the 3 significant variables, the model explains 69% of variation of the blood cholesterol level in the study sample. ($R^2=0.69$)

SUMMARY

Step 1: Data exploration (Descriptive Statistics)
Step 2: Scatter plots and Simple Linear Regression

} Exploring

Step 3: Variable selection

⇒ Preliminary main-effect model

Step 4: Checking interaction & multicollinearity^a

⇒ Preliminary final model

} Modeling

Step 5: Checking model assumptions & outliers^a

⇒ Final model

Step 6: Interpretation & data presentation

} Checking assumptions and interpretation

^a need remedial measures if problems are detected.

Categorical Independent Var.

Cautions:

It should be coded (0, 1) for dichotomous variable.

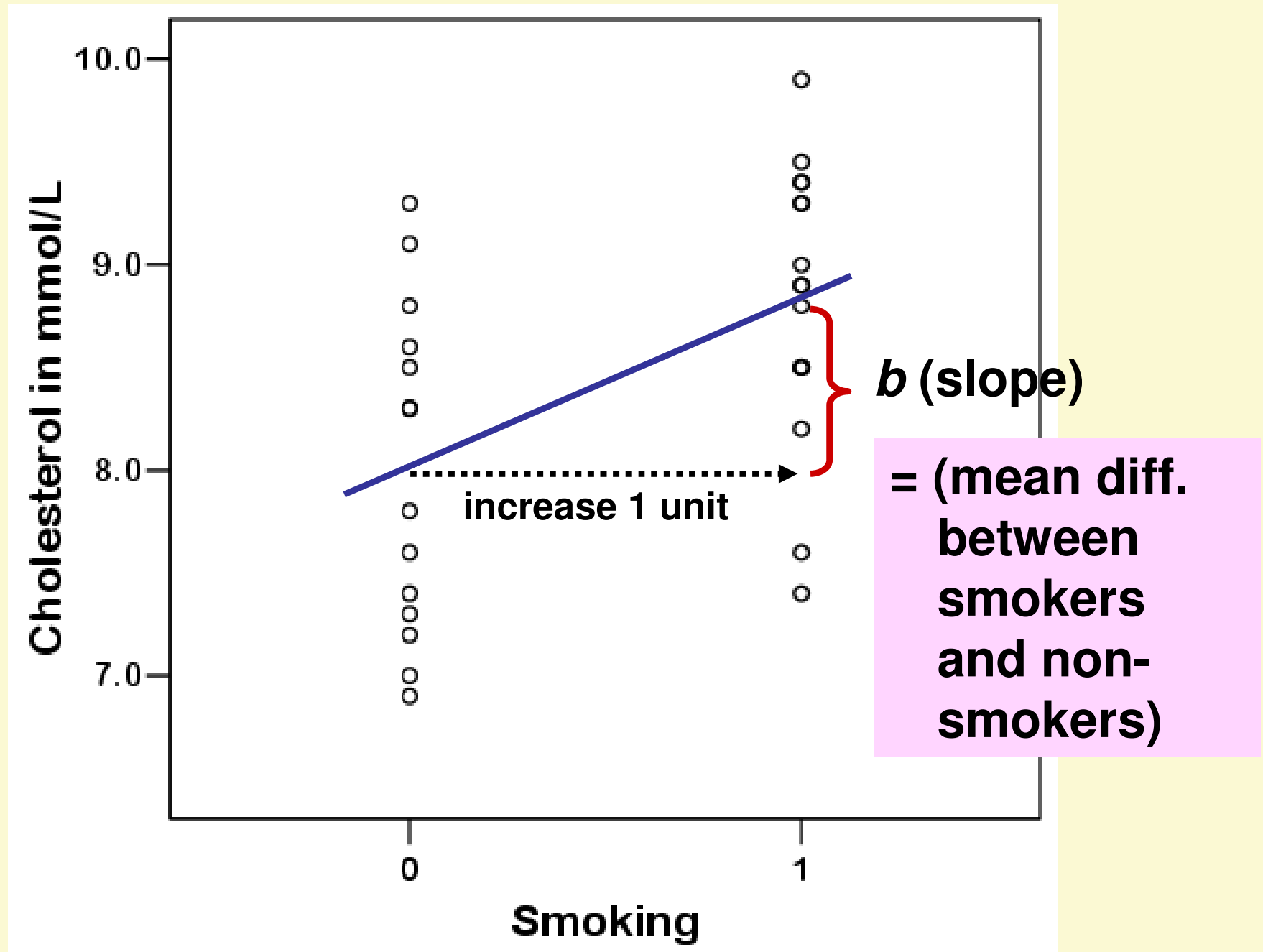
Example 1: sex (male=1, female=0)

It means we are comparing male against female (female as reference)

Example 2: smoking (smokers=1, non-smoker=0)

It means we are comparing smokers against non-smoker (non-smoker as reference)

Say, outcome is cholesterol, smoking as independent var., and we got $b=2.0$. It means smokers will have cholesterol level higher than non-smokers for 2.0 mmol/L.



Categorical Independent Var.

Cautions:

If you have more than 2 categories in categorical variable, we have to create Dummy Variables.

Example: Education level (no education=1; primary school level=2; secondary level=3)

Then, we need to create 2 dummy variables: (e.g. edu2 & edu3)

	edu2	edu3
No edu. →	0	0
Primary edu. →	1	0
Secondary edu. →	0	1

Here, reference is 'no education',

edu2 is comparing 'primary' against 'no edu', and

edu3 is comparing 'secondary' against 'no edu'.

Categorical Independent Var.

Example 2: Education level (no education=1; primary=2; secondary=3; tertiary=4)

Then, we need to create 3 dummy variables: (e.g. edu2 & edu3 & edu4)

	edu2	edu3	edu4
No edu. →	0	0	0
Primary edu. →	1	0	0
Secondary edu. →	0	1	0
Tertiary edu. →	0	0	1

Categorical Independent Var.

Cautions:

If you have more than 2 categories in categorical variable, we have to create Dummy Variables.

Example: **Agegp**: Age (<35)=1; Age (35-44)=2; Age (>=45)=3

Then, we need to create 2 dummy variables: (e.g. agegp2 & agegp3)

agegp		agegp2	agegp3
<35 (1) 'yg'	→	0	0
35-44 (2) 'older'	→	1	0
>=45 (3) 'eldest'	→	0	1

Here, reference is 'young',
agegp2 is comparing 'older'
against 'young', and
agegp3 is comparing 'eldest'
against 'young'.

Recode into Different Variables

- # chol
- # age
- # diet
- # exercise
- # se_stat



Numeric Variable -> Output Variable:

agegp -> agegp2

Output Variable Name:

agegp2

Label:

Old and New Values...

**‘Recode’
into different
variables**

Recode into Different Variables: Old and New Values

Old Value

Value:

System-missing

System- or user-missing

Range:

through

New Value

Value:

Copy old value(s)

Add

Change

Old --> New

1 --> 0

2 --> 1

3 --> 0

Recode into Different Variables

Numeric Variable -> Output Variable:

agegp --> agegp3

Output Variable Name: agegp3

Label:

Old and New Values...

Variables list: chol, age, diet, exercise, se_stat, agegp2

Recode into Different Variables: Old and New Values

Old Value

- Value: []
- System-missing
- System- or user-missing
- Range: [] through []

New Value

- Value: []
- Copy old value(s)

Old --> New

1	-->	0
2	-->	0
3	-->	1

Buttons: Add, Change

Linear Regression

Dependent:

Block 1 of 1

Independent(s):

- # se_stat
- # agegp2
- # agegp3

Method:

All variables including 2 age dummy variables

Variable selection procedure

5	(Constant)	8.529	.397		21.473	.000	7.738	9.3
	exercise	-.536	.064	-.559	-8.368	.000	-.664	-.4
	diet	.390	.053	.473	7.310	.000	.284	.4
	agegp3	.351	.149	.157	2.354	.021	.054	.6

SE is out, and only agegp3 is selected. However, agegp3 is part of age variable, and both dummy variables must be in the model (to complete as the age variable).

Linear Regression

Dependent: chol

Block 1 of 1

Independent(s): exercise, agegp2, agegp3

Method: Enter

SE out
Add agegp2

We have to force agegp2 to complete as the age-group variable.

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	!
		B	Std. Error	Beta			
1	(Constant)	8.445	.415		20.373	.000	
	diet	.391	.054	.474	7.302	.000	
	exercise	-.539	.064	-.562	-8.369	.000	
	agegp2	.114	.155	.062	.737	.464	
	agegp3	.439	.192	.197	2.291	.025	

a. Dependent Variable: chol

How to interpret 'b' of categorical variable?

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	!
		B	Std. Error	Beta			
1	(Constant)	8.445	.415		20.373	.000	
	diet	.391	.054	.474	7.302	.000	
	exercise	-.539	.064	-.562	-8.369	.000	
	agegp2	.114	.155	.062	.737	.464	
	agegp3	.439	.192	.197	2.291	.025	

a. Dependent Variable: chol

- ❑ There is no significant difference in cholesterol level between older age-group (35-44) and young group (<35) ($P=0.464$).
- ❑ However, the eldest group (≥ 45) have significantly higher cholesterol level than the young group (<35) ($P=0.025$).
- ❑ The eldest group (≥ 45) have 0.44 mmol/L higher cholesterol level than the young group (<35) (95% CI: 0.06, 0.82 mmol/L).



Questions & Answers